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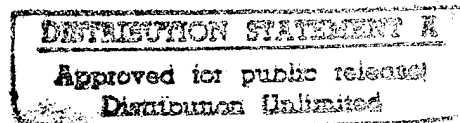
### ANALYSIS OF EDGE IMPACT STRESSES IN COMPOSITE PLATES

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prepared for  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
Washington, D. C. 20546

NASA Lewis Research Center, Cleveland, Ohio 44135  
Contract NGR 31-001-267

Topical Report  
May 1973 to May 1974



19960206 138

Department of  
Aerospace and  
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1. Report No. NASA CR-134720		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle Analysis of Edge Impact Stresses in Composite Plates				5. Report Date July 1974	
				6. Performing Organization Code	
7. Author(s) F. C. Moon, C. K. Kang				8. Performing Organization Report No. AMS Report No. 1141	
9. Performing Organization Name and Address  Princeton University Princeton, NJ 08540				10. Work Unit No.	
				11. Contract or Grant No. NGR 31-001-267	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546				13. Type of Report and Period Covered Topical Report May 1973 to May 1974	
				14. Sponsoring Agency Code	
15. Supplementary Notes Project Manager, C. C. Chamis Materials and Structures Division NASA-Lewis Research Center Cleveland, OH 44135					
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17. Key Words (Suggested by Author(s)) Fiber composite plates, high velocity impact, in-plane edge impact, edge protection, foreign object damage, displacement and stress waves, contact time, fast fourier transform, computer program				18. Distribution Statement  Unclassified, unlimited	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 73	
				22. Price* \$3.00	

\* For sale by the National Technical Information Service, Springfield, Virginia 22151

ABSTRACT

The in-plane edge impact of composite plates, with or without a protection strip, is investigated in this work. A computational analysis based on the Fast Fourier Transform technique is presented. The particular application of the present method is in the understanding of the foreign object damage problem of composite fan blades. However, the method is completely general, and may be applied to the study of other stress wave propagation problems in a half space. Results indicate that for the protective strip to be effective in reducing impact stresses in the composite the thickness must be equal or greater than the impact contact dimension. Also large interface shear stresses at the strip - composite boundary can be induced under impact.

## TABLE OF CONTENTS

	<u>Page</u>
Abstract	i
Table of Contents	ii
List of Tables	iii
List of Figures	iv, v
Introduction	1
I Basic equations for edge loading on an anisotropic plate	4
II Edge impact of plate with edge protection	11
III Numerical inversion	14
IV Results	20
Summary of results	23
References	24
Tables	25
Appendicies	
A. Determination of parameter $C_0$ in Laplace inversion	26
B. Flow chart of computer program	28
C. Notes on computer program	30
D. Program input and output.	32
Figures.	

LIST OF TABLES

	page
Table I Stress-Strain coefficients for 55% graphite fiber/epoxy matrix composite	25

LIST OF FIGURES\*

1. Geometry of in-plane edge impact of a composite plate with a protection strip.
2. Body wave speeds and Rayleigh wave speed versus fiber lay-up angle for 55% graphite fiber in epoxy matrix.
3. Branches and singular points in the complex Laplace plane.
4. Rayleigh wave for stress  $t_{11}$ , impact time 35  $\mu$ sec, impact length 2 cm, 55% graphite fiber/epoxy matrix  $0^\circ$  lay-up angle - no strip.
5. Rayleigh wave for stress  $t_{11}$ , impact time 35  $\mu$ /sec, impact length 2 cm, 55% graphite fiber/epoxy matrix  $15^\circ$  lay-up angle.
6. Level computer map in  $x_1$  - time plane for applied stress  $t_{33}$ ,  $0^\circ$  LAY-UP ANGLE on  $x_3 = 0$ .
7. Level computer map in  $x_1$  - time plane for stress wave  $t_{11}$ ,  $0^\circ$  Lay-up angle on  $x_3 = 0$ .
8. Decrease of impact stress with distance from the impact edge.
9. Stress  $t_{33}$  versus time for normalized depths  $x_3 = 0, 1, 2$ .
10. Maximum impact stress levels versus layup angles for 55% graphite fiber/epoxy matrix composite.
11. Effect of layup angle on distribution of stress  $t_{33}$  versus  $x_1$  at a normalized depth  $x_2 = 1$  for 55% graphite fiber/epoxy matrix composite.
- 12a Effect of steel edge strip thickness on impact stresses at the strip edge interface for 55% graphite fiber/epoxy matrix composite,  $\pm 15^\circ$  layup angle.
- 12b Effect of steel edge strip thickness on impact stresses at the strip edge interface for 55% graphite fiber/epoxy matrix composite  $\pm 45^\circ$  layup angle.
13. Effect of Aluminum edge strip thickness on impact stresses at the strip edge interface for 55% graphite fiber/epoxy matrix composite  $0^\circ$  layup angle.
14. Effect of steel edge strip thickness on  $t_{33}$  stress distribution versus along edge interface for normalized thicknesses  $b/a = 0.1, 1.0$ , 55% graphite fiber/epoxy matrix composite.

\*Except for Figure 6, a contact time of 35  $\mu$ sec and contact length of 2 cm was used in these calculations.

15. Effect of steel edge strip thickness on  $t_{11}$  stress distribution versus along edge interface for normalized thicknesses  $b/a = 0.1, 1.0, 55\%$  graphite fiber/epoxy matrix composite.
16. Effect of steel edge strip thickness on edge interface shear stress distribution for normalized edge thickness  $b/a = 0.25, 2.0, 55\%$  graphite fiber/epoxy matrix composite.



## INTRODUCTION

This report is part of a continuing effort by NASA to understand the basic mechanics of foreign object impact of composite materials of particular interest are damage resistant designs of jet engine fan blades under hail or bird impact. In previous reports the central or normal impact response of composite plates was examined [1]. In this report the mechanics of edge impact of composite plates are examined. This is schematically illustrated in Figure 1.

The basic approach to the study of impact of composite plates in this program has been to examine the stress waves generated by the impact forces. For central impact of plates it has been shown that in addition to wave propagation across the plate thickness, bending and extensional waves propagate away from the impact site. The stresses associated with these waves have been studied without considering the effect of boundaries such as the free or clamped edges of a fan blade. This simplification has been made on the premise that for short impact times e.g. less than  $10^{-4}$  sec. few edge reflections have taken place, and that the highest stresses occur at the impact site. However, for edge impact, the boundary conditions greatly affect the nature of the wave mechanics.

Edge waves in solids have been studied extensively in seismology. The principal phenomenon is the entrapment of wave energy in a layer near the surface. This surface wave is known as a Rayleigh wave and travels at a velocity below the shear velocity for isotropic solids. For plates, however, two types of edge waves can occur as shown in Figure 2a. For impact transverse to the plate, flexural edge waves can occur. For in-plane, Rayleigh type edge waves are generated. In this report only in-plane edge waves will be discussed.

Wave type solutions to the equations of elastodynamics of an orthotropic plate which exponentially decay away from the edge ( $X_3$  direction) and propagate along the edge ( $X_1$  direction) can be found provided the edge wave velocity satisfies the equation (Reference 9).

$$\rho v^2 + \left[ \begin{array}{cc} C_{55} & -\rho v^2 \\ C_{33} & C_{55} \end{array} \begin{array}{c} (C_{11} - \rho v^2) \\ C_{11} \end{array} \right]^{1/2} [C_{13}^2 - C_{33}(C_{11} - \rho v^2)] = 0$$

where  $\rho$  is the mass density of the plate and  $C_{ij}$  are the effective plate elastic constants (denoted by  $\hat{C}_{ij}$  in Reference 1). It can be shown that one real root lies in the interval

$$0 < \rho v^2 < C_{55}$$

Thus, the edge or Rayleigh wave speed is less than the shear speed in this direction  $[C_{55}/\rho]^{1/2}$ .

Changing the layup angle will affect the elastic constants  $C_{ij}$  and hence, change the value of the edge wave velocity. The results of this calculation are shown in Figure 2b where the  $C_{ij}$  are obtained from Reference 7. The edge wave speed seems to obtain a maximum between  $\pm 15$  and  $\pm 30^\circ$  layup angle which is below the extentional wave speeds labelled "dilatational" and "shear" in Figure 2b and which is greater than the bending wave velocity.

In order to prevent damage to composite fan blades under foreign object impact leading edge protection has been used. (See e.g. Ref. (8)). This usually consists of a strip of metal attached to the leading edge of the fan blade. To model the effects of this impact protection strip, the in-plane edge impact of an anisotropic plate, with a beam-strip attached to the impact edge, has been studied (see Figure 1). It will be shown later that the strip will decrease the tensile stress along the edge while producing shear stress between the strip and the plate edge.

# I. BASIC EQUATIONS FOR EDGE LOADING ON ANISOTROPIC PLATE

Let the plate be the half-space  $x_3 > 0$ . The equations of motion are given by [1] as:

$$C_{11} u_{1,11} + C_{55} u_{1,33} + (C_{13} + C_{55}) u_{3,13} = \rho u_{1,tt} \quad (1.1)$$

$$C_{33} u_{3,33} + C_{55} u_{3,11} + (C_{13} + C_{55}) u_{1,13} = \rho u_{3,tt}$$

here we have employed  $C_{11}, C_{13}, C_{33} \dots$  to denote  $\hat{C}_{11}, \hat{C}_{13}, \hat{C}_{33} \dots$  of [1]. The in-plane motion is assumed to be independent of the bending deformation. With the loading condition shown in Figure 1, the boundary conditions are (without protection strip):

$$t_{13}(x_1, 0, t) = C_{55} (u_{1,3} + u_{3,1}) \Big|_{x_3=0} = 0 \quad (1.2)$$

$$t_{33}(x_1, 0, t) = [C_{33} u_{3,3} + C_{13} u_{1,1}] \Big|_{x_3=0} = p f(x_1) g(t)$$

The following nondimensional parameters are used;

$$C_{11}^* = C_{11}/C_{66}, \quad C_{13}^* = C_{13}/C_{66}, \quad C_{33}^* = C_{33}/C_{66}, \quad C_{55}^* = C_{55}/C_{66}, \quad P_0^* = p/C_{66} \quad (1.3)$$

$$u_1^* = u_1/\ell, \quad u_3^* = u_3/\ell, \quad x_1^* = x_1/\ell, \quad x_3^* = x_3/\ell, \quad t^* = t \sqrt{C_{66}/\rho}/\ell \quad (1.4)$$

$\ell$  is a length parameter.  $C_{66}$  is a typical elastic constant. In what follows the equations will be assumed to be nondimensionalized using (1.3) and (1.4).

To obtain the solution, we employ transform methods. Define:

$$F(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ik_1 x_1} f(x_1) dx_1$$

as the Fourier transform of  $f(x)$  where we assume that

$$f(x_1) \Big|_{-\infty}^{\infty} = \frac{\partial}{\partial x_1} f(x_1) \Big|_{-\infty}^{\infty} = 0$$

Then

$$F\left(\frac{\partial}{\partial x_1} f\right) = ik_1 F(f), \quad F\left(\frac{\partial^2}{\partial x_1^2} f\right) = -k_1^2 F(f)$$

Also define

$$\mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t) dt$$

as the Laplace transform. The initial conditions are assumed to be,

$$u_1(x_1, x_3, 0) = \frac{\partial}{\partial t} u_1(x_1, x_3, 0) = 0 \quad (1.5)$$

$$u_3(x_1, x_3, 0) = \frac{\partial}{\partial t} u_3(x_1, x_3, 0) = 0$$

Letting  $\mathcal{L}[F(u_1)] = \bar{u}_1(k, x_3, s)$ ,  $\mathcal{L}[F(u_3)] = \bar{u}_3(k, x_3, s)$ , we have the following transformed equation:

$$C_{11}(-k^2) \bar{u}_1 + C_{55} \bar{u}_{1,33} + (C_{13} + C_{55}) (ik_1) \bar{u}_{3,3} = +s^2 \bar{u}_1 \quad (1.6)$$

$$C_{33} \bar{u}_{3,33} + C_{55}(-k^2) \bar{u}_3 + (C_{13} + C_{55}) (ik_1) \bar{u}_{1,3} = +s^2 \bar{u}_3$$

and at  $x_3 = 0$ , the transformed boundary conditions become

$$\bar{u}_{1,3} + ik_1 \bar{u}_3 = 0 \quad (1.7)$$

$$C_{33} \bar{u}_{3,3} + C_{13} ik_1 \bar{u}_1 = -p_0 \overline{f(k_1)g(s)}$$

Since we are expecting surface wave propagation, we seek the solution of (1.6), (1.7) in the forms:

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} e^{-p(k,s)x_3} \quad (\text{real } p) \geq 0 \quad (1.8)$$

Therefore, the equations for  $\phi_1, \phi_2$  are:

$$\begin{bmatrix} -s^2 - C_{11} k_1^2 + p^2 C_{55} & -ik_1 p(C_{13} + C_{55}) \\ -ik_1 p(C_{13} + C_{55}) & -s^2 - k_1^2 C_{55} + p^2 C_{33} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0 \quad (1.9)$$

or for a non-trivial solution,

$$\begin{aligned} \det = C_{33} C_{55} p^4 + [C_{33} (-s^2 - C_{11} k_1^2) + C_{55} (-s^2 - C_{55} k_1^2) + k_1^2 (C_{13} + C_{55})^2] p^2 \\ + (s^2 + C_{11} k_1^2) (s^2 + C_{55} k_1^2) = 0 \end{aligned} \quad (1.10)$$

We will choose the  $p$ 's with positive real parts to ensure the decay in  $x_3$  direction of the surface wave. Let the solutions be  $p = p_1, p_2$ , therefore, we have

$$p = p_1: \phi_1^{(1)} \equiv C_1(k_1, s), \quad \phi_2^{(1)} = - \frac{i[-s^2 - C_{11} k_1^2 + C_{55} p_1^2]}{k_1 p_1 (C_{13} + C_{55})} \phi_1^{(1)} \equiv \psi_{31} C_1 \quad (1.11)$$

$$p = p_2: \phi_1^{(2)} \equiv C_2(k_1, s), \quad \phi_2^{(2)} = \psi_{32} C_2 \equiv - \frac{i[-s^2 - C_{11} k_1^2 + C_{55} p_2^2]}{k_1 p_2 (C_{13} + C_{55})} \phi_1^{(2)}$$

Therefore, the displacements have the forms:

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = C_1(k_1, s) \begin{bmatrix} 1 \\ \psi_{31} \end{bmatrix} e^{-p_1 x_3} + C_2(k_1, s) \begin{bmatrix} 1 \\ \psi_{32} \end{bmatrix} e^{-p_2 x_3} \quad (1.12)$$

$C_1, C_2$  are determined from the boundary conditions (1.7), or

$$\begin{bmatrix} -p_1 + ik_1 \psi_{31} & -p_2 + ik_1 \psi_{32} \\ -p_1 C_{33} \psi_{31} + ik_1 C_{13} & -p_2 C_{33} \psi_{32} + ik_1 C_{13} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -p_0 \overline{fg} \end{bmatrix} \quad (1.13)$$

Here, the determinant,  $\Delta(p_1, s)$ , given by,

$$\begin{aligned} \Delta &= (ik_1 \psi_{31} - p_1) (ik_1 C_{13} - p_2 C_{33} \psi_{32}) \\ &\quad - (ik_1 \psi_{32} - p_2) (ik_1 C_{13} - p_1 C_{33} \psi_{31}) \end{aligned} \quad (1.14)$$

must be non-zero to ensure a solution. ( $\Delta=0$  gives the Rayleigh poles of the system, which correspond to a free surface). Therefore,

$$C_1 = \frac{1}{\Delta} [ik_1 \psi_{32} - p_2] \overline{fg} p_0 \quad (1.15)$$

$$C_2 = \frac{1}{\Delta} [p_1 - ik_1 \psi_{31}] \overline{fg} p_0$$

and the physical displacements  $u_1(x_1, x_3, t), u_3(x_1, x_3, t)$  are obtained by inverting their transforms.



From the stress-strain relations

$$\begin{aligned} t_{13} &= C_{55}(u_{1,3} + u_{3,1}), \quad t_{11} = C_{11}u_{1,1} + C_{13}u_{3,3}, \\ t_{33} &= C_{33}u_{3,3} + C_{13}u_{1,1} \end{aligned} \quad (1.16)$$

we obtain the transforms of the stresses

$$\begin{aligned} \bar{t}_{13} &= C_{55}\{(-p_1 + ik_1 \psi_{31}) C_1 e^{-p_1 x_3} \\ &\quad + (-p_2 + ik_1 \psi_{32}) C_2 e^{-p_2 x_3}\} \\ \bar{t}_{11} &= (C_{11}ik_1 - C_{13}p_1 \psi_{31}) C_1 e^{-p_1 x_3} \\ &\quad + (C_{11}ik_1 - p_2 \psi_{32} C_{13}) C_2 e^{-p_2 x_3} \\ \bar{t}_{33} &= (-p_1 \psi_{31} C_{33} + ik_1 C_{13}) C_1 e^{-p_1 x_3} \\ &\quad + (-p_2 \psi_{32} C_{33} + C_{13}ik_1) C_2 e^{-p_2 x_3}, \end{aligned} \quad (1.17)$$

and the physical stresses can be obtained by inversion.

The particular forcing function employed is

$$f(x_1) g(t) = \left[ 1 - \left( \frac{x_1}{a} \right)^2 \right] \sin \frac{\pi t}{\tau_0} \quad 0 < t < \tau_0 \quad |x_1| \leq a$$

$$= \left[ 1 - \left( \frac{\ell}{a} \right)^2 x_1^{*2} \right] \sin (\pi t^*/\tau_0^*) \quad (1.18)$$

where  $\tau_0^* \equiv \ell / \sqrt{C_{66} / \rho}.$

Here  $a$  is a length measuring the impact area, and  $\tau_0$  is the contact time [1]. The transform of this particular forcing function, in non-dimensional form becomes,

$$\overline{fg} = + \frac{4}{k_1^2 (a/\ell)^2} \left[ - \frac{a}{\ell} \cos k_1 \left( \frac{a}{\ell} \right) + \sin k \left( \frac{a}{\ell} \right) / k \right] \frac{\pi \tau_0^* (1 + e^{-s \tau_0^*})}{\pi^2 + \tau_0^{*2} s^2}.$$

(1.19)

## II. EDGE IMPACT OF PLATE WITH EDGE PROTECTION

To prevent failure of composite fan blades under impact forces, leading edge protective strips have been employed. In practice, these strips of stainless steel are wrapped around the leading edge. To model this device, we consider a beam bonded to the edge of an anisotropic plate (Figure 1). The effect of the beam will be to thwart the force of impact, thereby decreasing the normal stresses in the composite. However, we shall show that with such a reduction in normal stress, sizeable interface shear stresses can be induced.

With the introduction of a beam of thickness  $b$  on the edge of the composite plate, the Rayleigh wave behavior will depend on the ratio of the wavelength to thickness ratio of each Fourier component in the  $x_1$  direction. Thus one should expect the Rayleigh wave speed to vary with  $b/a$ , the thickness to impact footprint ratio. In addition the Rayleigh wave will become distorted as it propagates.

To solve the edge strip problem the solution in the composite plate follows the same procedure as the no-strip case except for the boundary conditions on the edge. In place of the zero stress conditions on the edge we relate the edge stresses  $t_{33}$ ,  $t_{13}$  to the motion of the beam strip. If one considers a small element of the beam-strip along the  $x_1$  direction, the momentum balance equations in the  $x_1$ ,  $x_3$  directions become, (for a plate of unit thickness)

$$\rho b \frac{\partial^2 U}{\partial t^2} = E b \frac{\partial^2 U}{\partial x_1^2} + t_{13} \quad (2.1)$$

$$\rho b \frac{\partial^2 W}{\partial t^2} = -EI \frac{\partial^4 W}{\partial x_1^4} + I_\rho b \frac{\partial^4 W}{\partial x_1^2 \partial t^2} + \frac{b}{2} \frac{\partial t_{13}}{\partial x_1} + t_{33} + p f g \quad (2.2)$$

In these equations  $U, W$  are the  $x_1, x_3$  displacements of the beam element at the half thickness, and  $t_{33}, t_{13}$  are the interface stresses.

We choose the compatibility conditions between the beam and plate displacements

$$W = u_3, \text{ on } x_3 = 0. \quad (2.3)$$

$$U = u_1 + \frac{b}{2} \frac{\partial u_3}{\partial x_1}, \text{ on } x_3 = 0 \quad (2.4)$$

In the above equations  $b$  is the depth of the strip,  $E, I, I_\rho$  are respectively the Young's modulus, moment of inertia and rotary inertia. Also  $p f(t) g(x_1)$  is the edge loading now applied to the outer protective strip surface.

The equations for the plate remain as in the free edge case and a solution is obtained by taking a Laplace transform on time and a Fourier transform on the space variable  $x_1$ . With nondimensionalization the solution in the plate is assumed in the form of (1.12). The transform of the plate displacements are

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ \psi_{31} \end{bmatrix} e^{-p_1 x_3} + C_2 \begin{bmatrix} 1 \\ \psi_{32} \end{bmatrix} e^{-p_2 x_3} \quad (2.5)$$

where  $p_1, p_2$  are defined in (1.10) and  $\psi_{31}, \psi_{32}$  are given in (1.11).  $C_1, C_2$  are determined from the edge boundary conditions. However, in place of the free edge conditions (1.2) we use the equations of motion for the strip (2.1),

(2.2).  $C_1, C_2$  are then solutions to the algebraic equations

$$\begin{bmatrix} G_1 & H_1 \\ G & H \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -p_o^* \bar{f}g \end{bmatrix} \quad (2.6)$$

where

$$G_1 = -k^2 Ew + p_1 C_{55} - \rho w s^2 - \psi_{31} (ik^2 w^2 E/2 + ik C_{55} + \rho w^2 s^2 ik/2)$$

$$H_1 = -k^2 Ew + p_2 C_{55} - \rho w s^2 - \psi_{32} (ik^3 w^2 E/2 + ik C_{55} + \rho w^2 s^2 ik/2)$$

$$G = -ik C_{13} + ik p_1 C_{55}/2 + \psi_{31} (p_1 C_{33} + Ew^2 k^4/12 + wk^2 C_{55}/2 + k^2 s^2 \rho w^3/12 + \rho w s^2)$$

$$H = -ik C_{13} + ik p_2 C_{55}/2 + \psi_{32} (p_2 C_{33} + Ew^2 k^4/12 + wk^2 C_{55}/2 + k^2 s^2 \rho w^3/12 + \rho w s^2) \quad (2.7)$$

where  $w = b/\ell$ ,  $\rho$ ,  $E$ , are nondimensionalized quantities and  $p_1, p_2$ ,

$\psi_{31}, \psi_{32}$  are defined in (1.11).

### III. NUMERICAL INVERSION

The inversions are accomplished by the Fast Fourier Transform (FFT) techniques [2], which consists of a transformation from Laplace to Fourier transforms, and a two-dimensional numerical inversion using the usual FFT algorithm. Notice the Laplace inversion formula

$$f(t) = \frac{1}{2\pi i} \int_{\Gamma}^{C+i\infty}_{C-i\infty} f(s) e^{st} ds$$

Set  $s = C + i\alpha$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(C+i\alpha) e^{Ct} e^{i\alpha t} d\alpha \quad (3.1)$$

where  $C$  and  $\alpha$  are both real, and  $C$  is greater than the largest real part of all singularities of  $f(s)$ . Numerically, the double Fourier transform (or inversion) has the following form [1]:

$$f(x,t) \approx \frac{K_x K_t}{\pi^2 NM} e^{-i[K_x(1-\frac{1}{N})x + K_t(1-\frac{1}{M})t]} \sum_{I=1}^N \sum_{J=1}^M \bar{f}(I,J) e^{2\pi i [\frac{(I-1)x}{N} + \frac{(J-1)t}{M}]} \quad (3.2)$$

where  $N, M$  are the number of points in  $x$  and  $t$  direction respectively, and  $K_x, K_t$  are the corresponding half-frequency range.

For the present problem, the determination of  $C$  is through the following considerations:

The form of inversion integrals are, in general,

$$I = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} \frac{F(k_1, s)}{\Delta(k_1, s)} e^{-px_3} e^{st} ds \quad p = p(k_1, s), \operatorname{Re}(p) \geq 0 \quad (3.3)$$

and  $\Delta$  is given by (1.14). It is easy to see the singularities of the integral of (2.3) are:

a) Poles at  $\Delta = 0$ ,

$$\Delta = (p_2 - p_1) i k_1 (C_{13} - \psi_{31} \psi_{32} C_{33}) + (\psi_{32} - \psi_{31}) (k_1^2 C_{13} + p_1 p_2 C_{33}) = 0 \quad (3.4)$$

implies

$$\begin{aligned} & (p_2 - p_1) [C_{13} k_1^2 p_1 p_2 (C_{13} + C_{55})^2 + C_{33} (C_{55} p_1^2 - s^2 - C_{11} k_1^2) (C_{55} p_2^2 - s^2 - C_{11} k_1^2)] \\ & - (C_{13} + C_{55}) [p_1 (C_{55} p_2^2 - s^2 - C_{11} k_1^2) - p_2 (C_{55} p_1^2 - s^2 - C_{11} k_1^2)] (k_1^2 C_{13} + p_1 p_2 C_{33}) \\ & = 0 \end{aligned} \quad (3.5)$$

Interpretation of this condition is best understood  
for the particular case of an isotropic material, i.e. for

$$C_{11} = \lambda + 2\mu = C_{33}, \quad C_{13} = \lambda, \quad C_{55} = \mu,$$

$$\text{choose } C_{66} = C_{55} = \mu$$

It is easy to see that from (1.10), (1.11)

$$\det = [\mu p^2 - (\mu k_1^2 + \rho s^2)] \{ (2\mu + \lambda) p^2 - [(\lambda + 2\mu) k_1^2 + \rho s^2] \}$$

$$p_1^2 = (\mu k_1^2 + \rho s^2) / \mu, \quad p_2^2 = [(\lambda + 2\mu) k_1^2 + \rho s^2] / (\lambda + 2\mu)$$

$$\psi_{31} = ik_1 / p_1, \quad \psi_{32} = -p_2 / ik_1,$$

Here  $[\mu/\rho]^{1/2} \equiv v_s$ , is the shear wave speed and  $[(\lambda + 2\mu)/\rho]^{1/2} v_p$ , is the longitudinal or pressure wave speed for isotropic materials.



Thus for the isotropic case that,  $\Delta = 0$  implies

$$4\mu p_2 + (p_1 + \frac{k^2}{p_1}) [\lambda + \frac{p_2^2}{k^2} (\lambda + 2\mu)] = 0 \quad (3.6)$$

where  $p_1, p_2$  are defined above. This is the equation for the Rayleigh wave speed  $v_R \equiv (is/k)^{1/2}$  which is found as a real root of (3.6) (see e.g. [3]).

For the case  $\lambda = \mu$  (Poissons ratio = 0.25),  $v_R = 0.919 v_s$ .

For the anisotropic case a computational scheme to calculate the zeroes of (3.5) has been written, (Figure 2).

b) Branch points: The branch points of the integrand in (3.3) are the same as those of the functions  $p_1(k_1, s), p_2(k_1, s)$ , they are:

$$\begin{aligned} \text{i) } p_1 = 0, \text{ or } p_2 = 0 \text{ which implies } (C_{11} k^2 + \rho s^2) (C_{55} k^2 + \rho s^2) = 0 \\ \text{i.e., } k/s = \pm i\sqrt{\rho/C_{55}}, \pm i\sqrt{\rho/C_{11}} \text{ pure imaginary.} \end{aligned}$$

These correspond to longitudinal and shear wave speeds for an isotropic material

$$\text{ii) } p_{1,2} = \pm \sqrt{-B \pm \sqrt{B^2 - 4AC}} / 2A \quad (3.7)$$

with  $A = C_{33} C_{55} > 0$

$$B = -\rho s^2 (C_{33} + C_{55}) + k^2 [(C_{13} + C_{55})^2 - C_{11} C_{33} - C_{55}^2]$$

$$C = (C_{11} k^2 + \rho s^2) (C_{55} k^2 + \rho s^2)$$

These branch points are those values of  $s/k$  which render  $B - 4AC = 0$ , and are branch points of second order.<sup>[4]</sup> The distribution of these points is shown in Figure 3. It has been shown [5] that the contribution of these branch points to the value of the integrals (3.3) is important only when one considers the multi-reflected and refracted waves in layered media, or when the position of interest is very close to the impact origin. In this study we were more concerned with how the energy is propagated away from the impact point, which is mainly associated with surface waves, thus we ignored the contribution of these branch points.<sup>[5]</sup>

The contour of integration for the Laplace inversion is as shown in Figure 3. Notice the branch cuts are extended to negative infinity, in accordance with the requirement that  $C > \max$  real part of the singularities. The requirement that  $\text{real}(p) \geq 0$  also determines the correct sheet of the Riemann surfaces. Numerically, since the branch points are located at  $s/k = \text{constant}$ , as  $k$  gets large  $C$  should be large, and the factor  $e^{ct}$  in the Laplace inversion expression will rise sharply to an unmanageable size. Since, in the last paragraph, we have noticed the contribution of the branch points is unimportant, a path  $\Gamma_2$  is chosen to replace  $\Gamma_1$  by the Cauchy's integral theorem. Notice the advantage of integration along  $\Gamma_2$  is that  $C_0$  is a positive constant independent of  $k$ . The determination of optimal  $C_0$  is discussed in [2]. Here, in order to minimize the aliasing and round off errors in numerical computations simultaneously, we choose

$$C_o = \frac{2}{3M\lambda_t} \ln (\hat{g}/\hat{r}) \quad (3.9)$$

where  $\hat{g} = p_o/C_{66}$  ,  $\hat{r} = 10^{-6} \times \frac{1}{\lambda_t}$  .  $\lambda_t$  is a small time interval, less

than the impact contact time. Further details are given in Appendix A.

#### IV. RESULTS

A computer program has been written to calculate the stresses in a plate with an elastic beam on one edge under a transient impact load distribution along the edge. A program description, flow charts, input data formats, and sample printout of the program are contained in the appendices to this report. In this section we will summarize some of the results obtained from this computer program. These results were calculated for an anisotropic plate with effective elastic constants of 55% graphite fiber/epoxy matrix composite obtained from Reference 7 and summarized in the Table.

##### No-strip case:

The Rayleigh wave can be seen in the stress  $t_{11}$  on the edge as shown in Figures 4, 5 for a graphite fiber-epoxy composite for layup angles  $0, \pm 15^\circ$ . After the initial contact time, the stress is observed to propagate with little change at a speed near the calculated Rayleigh speed (Figure 2). This wave can be observed in the computed output in the space-time ( $x_1, -t$ ) plane Figure 7, as a band of non-zero values along a diagonal from the upper left to the lower right corner of the  $x_1, -t$  plane. Caution is urged in using this program since spurious waves can enter the calculations due to the periodic nature of the finite numerical Fourier Transform. These spurious waves are data bands which lie along diagonals from upper right to lower left. In other words, only disturbances emanating from the impact source in the upper left corner of the  $x_1 -t$  plane of Figure 7 should be valid. A computer map of the space time history of the edge impact stress is shown in Figure 6. A contact time of 35  $\mu\text{sec}$  and contact length 2 cm was used in these calculations.

As is characteristic of surface wave effects, the stresses due to impulsive loading on the edge decrease with distance from the edge. This is shown in Figure 8 for two different layup angles. The normal stress  $t_{33}$  appears to decrease to about 1/4 of its value on the surface at a depth equal to one half of the loading length  $a$ . The rate of decay from the edge depends on the layup angle. Another characteristic of edge impact is the development of tension in the normal stress  $t_{33}$  under the impact point. This is shown in Figure 9. Thus while the compression part decreases with distance from the edge a tension tail develops in the wave.

The effect of layup angle on the impact stresses can be seen in Figures 10, 11. The stress  $t_{11}$  at the edge is larger than the impact pressure and decreases as the layup angle goes from  $0^\circ$  to  $\pm 45^\circ$  (Figure 10).

Below the surface or edge, the peak normal stress  $t_{33}$  at  $x_1 = 0$  is a minimum for layup angles near  $\pm 30^\circ$ , while the shear stress  $t_{13}$  increases as the fiber angle goes from  $0^\circ$  to  $\pm 45^\circ$  (Figure 10).

An unexpected result is the shift of the maximum normal stress  $t_{33}$  to points off the impact axis  $x_1 = 0$  for layup angles greater than about  $\pm 30$ . A pronounced peak in  $t_{33}$  versus  $x_1$  beyond the impact pressure foot print, can be seen in Figure 11 for  $\pm 45^\circ$  layup angle.

#### Impact protection strip case:

The effect of bonding an edge impact protection strip to the half plane is shown in Figures 12-16. In Figures 12, 13, the increase in the thickness of a steel strip produces a decrease in the interface stresses

$t_{11}$ ,  $t_{33}$  but creates an interface shear stress at the strip-composite interface. This shear stress reaches a maximum for strip thickness less than the impact footprint length and decreases for greater strip thicknesses. Thus, if the strip is too thin, debonding can occur under impact due to induced interface shear.

In Figure 14 one can see that increasing the strip thickness decreases the peak normal stress  $t_{33}$  and redistributes the load over a longer length under the strip. However, while the peak compression stress is decreased by the strip, tension is created which could also produce debonding of the strip from the composite.

For the no-strip case a Rayleigh wave was seen to propagate along the edge relatively unperturbed (Figures 4, 5). With the strip present, (Fig.15) this wave becomes distorted as time increases. In fact, the beam-strip boundary conditions introduce dispersion in the edge waves which make the Rayleigh edge wave velocity dependent in the effective wave length of the disturbance.

Finally in Figure 16 shear stress distributions along the strip-composite interface are shown for a thickness near the shear peak ( $b/a = 0.25$ ) and another for  $b/a = 2.0$ . In the latter case the shear is distributed over a larger length resulting in a lower peak stress. Also the strip delays the time of maximum shear from  $1/2$  to  $3/4 T_0$

### Summary of Results

An analytical-numerical method has been developed to solve the response of a composite plate with a bonded edge strip to in-plane impact type forces on the edge. Results of computer simulations reveal the following:

- 1) Rayleigh edge waves can propagate away from the impact site with tension and compression up to values of the impact pressure, depending on layup angle.
- 2) Normal to the edge, the initial peak compression pulse decreases as it propagates into the plate but a tension tail develops as it propagates away from the impact site.
- 3) The edge stress  $t_{11}$  under impact is decreased as the fiber layup angle goes from  $0^\circ$  to  $\pm 45^\circ$ .
- 4) Protection strips of thickness less than half the impact length can develop large interface shear under impact.
- 5) The normal and edge stresses  $t_{33}$ ,  $t_{11}$  at the edge can be decreased significantly by protection strips of thickness greater than the half impact length.

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TABLE I. - STRESS-STRAIN COEFFICIENTS FOR 55 PERCENT GRAPHITE

FIBER-EPOXY MATRIX COMPOSITE

[All constants to be multiplied by  $10^6$  psi; data obtained from ref. 7.]

0° Layup					±15° Layup						
27.95	0.3957	0.3957	0	0	0	24.56	0.4000	1.986	0	0	0
1.170	0.4601	0	0	0	0	1.170	0.4558	0	0	0	0
1.170	0	0	0	0	0	1.374	0	0	0	0	0
0.3552	0	0	0	0	0	0.3552	0	0	0	0	0
0.7197	0	0	0	0	0	2.310	0	0	0	0	0
0.3552					0.3552					0.3552	
±30° Layup					±45° Layup						
16.48	0.4118	5.167	0	0	0	8.197	0.4279	6.758	0	0	0
1.170	0.4400	0	0	0	0	1.170	0.4279	0	0	0	0
3.093	0	0	0	0	0	8.179	0	0	0	0	0
0.3552	0	0	0	0	0	0.3552	0	0	0	0	0
5.491	0	0	0	0	0	7.082	0	0	0	0	0
0.3552					0.3552					0.3552	

# APPENDIX A

## The Determination of Parameter $C_0$

Consider the Laplace inversion of a function  $g(t)$  as

$$g(t) = \frac{1}{2\pi i} \int_{C_0 - i\infty}^{C_0 + i\infty} \bar{g}(s) e^{st} ds \quad (A.1)$$

and let  $s = C_0 + i\omega$ ,  $C_0$  and  $\omega$  are real. We can change (A.1) into the Fourier inversion formula

$$g(t) = \frac{1}{2\pi} e^{C_0 t} \int_{-\infty}^{\infty} \bar{g}(C_0 + i\omega) e^{i\omega t} d\omega \equiv e^{C_0 t} x(t) \quad (A.2)$$

which is then inverted by the Fast Fourier Transform technique. In the FFT scheme, a continuous function  $\bar{g}(C_0 + i\omega)$  is discretized and the infinite interval of integration is truncated. The error due to truncation depends on each problem but doesn't depend on  $C_0$ , thus in the determination of  $C_0$ , we will assume the truncation error is negligible. It is shown [2] that the discretizing of the transform in one domain will cause aliasing error in the other domain, e.g. sampling  $\bar{g}$  at  $N$  points in a frequency interval,  $0 < \omega < \Omega$ , will produce a transformed function  $x_p(t)$  which is periodic and which differs substantially from  $x(t)$  for large enough  $t$ . For even  $x(t)$  this difference can be shown [2] to be given by,

$$x_p(t) \approx x(t) + x(t-T)$$

for  $0 < t < T/2$  where  $T = N/\Omega$ .

It has been shown [2] also that the aliasing error is approximated by  $E_a(t) = e^{-C_o(T-2t)} g(T-t)$  for the Laplace inversion (A.2).

Notice the aliasing error is a decreasing function of  $C_o$ .

The other source of error is of course the round off error in computation. Since we multiply the resulting  $x(t)$  by  $e^{C_o t}$  to get  $g(t)$ , the rounding error is of the form

$$Er(t) = e^{C_o T} r(t).$$

The error bounds are then

$$\left. \begin{aligned} \epsilon_1 &= |\text{Max } E_a(t)| = e^{-C_o(T-2\tau)} \text{Max}|g(T-t)| \\ \epsilon_2 &= |\text{Max } Er(t)| = e^{C_o\tau} \text{Max}|r(t)| \end{aligned} \right\} 0 \leq t \leq \tau$$

Equating  $\epsilon_1$  and  $\epsilon_2$ , the optimal  $C_o$  is then

$$C_o = \frac{\ln (\text{Max } g(T-t) / \text{Max } r(t))}{T-\tau}$$

Choosing  $\tau = T/4$ , therefore

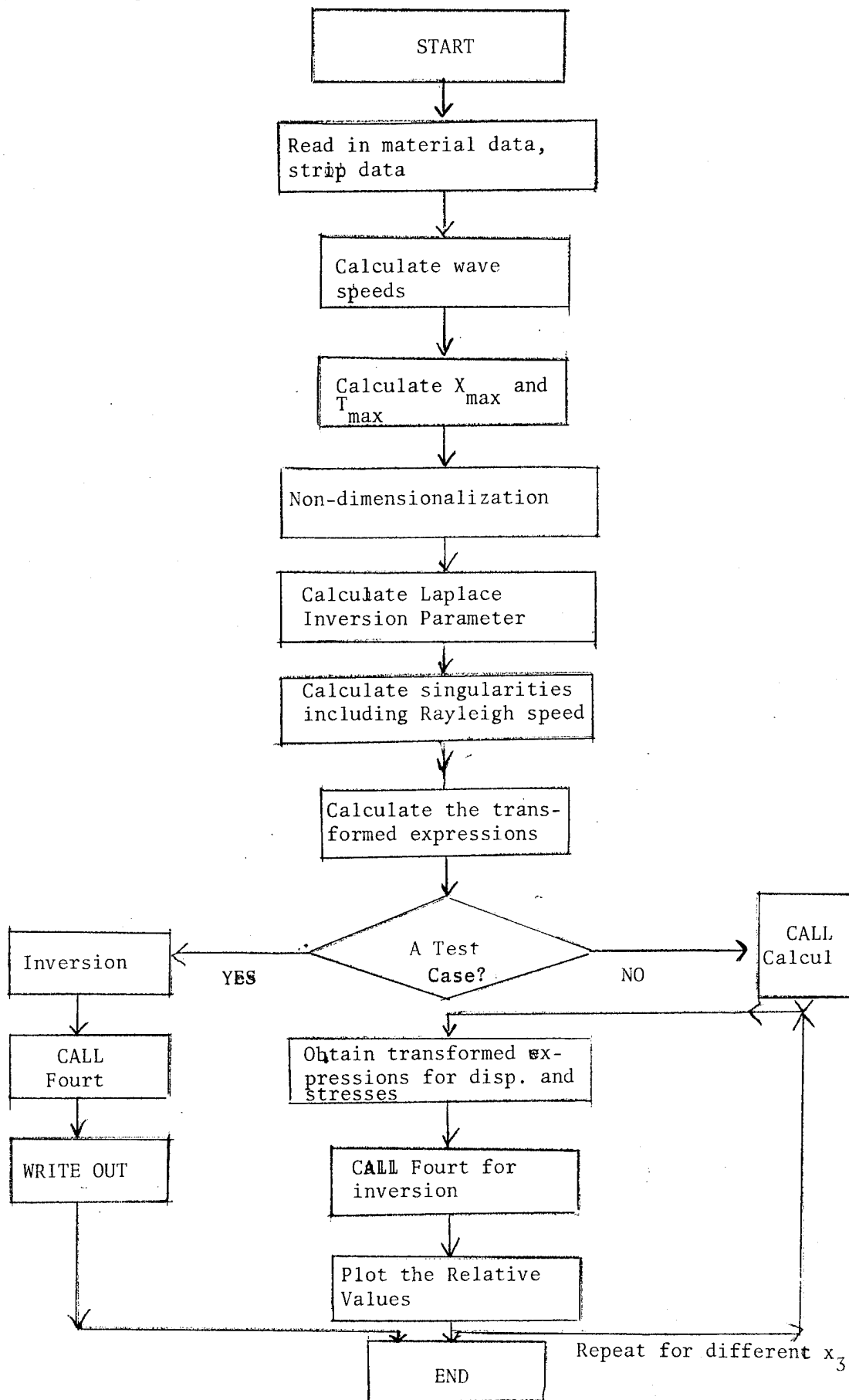
$$C_o = \frac{4}{3T} \ln(\hat{g}/\hat{r}), \quad \hat{g} \equiv \text{Max } g(T-t), \quad \hat{r} \equiv \text{Max } r(t)$$

Notice, empirically,  $\hat{r} \approx 2 \frac{N}{T} 10^{-6}$  on single precision IBM 360 systems.

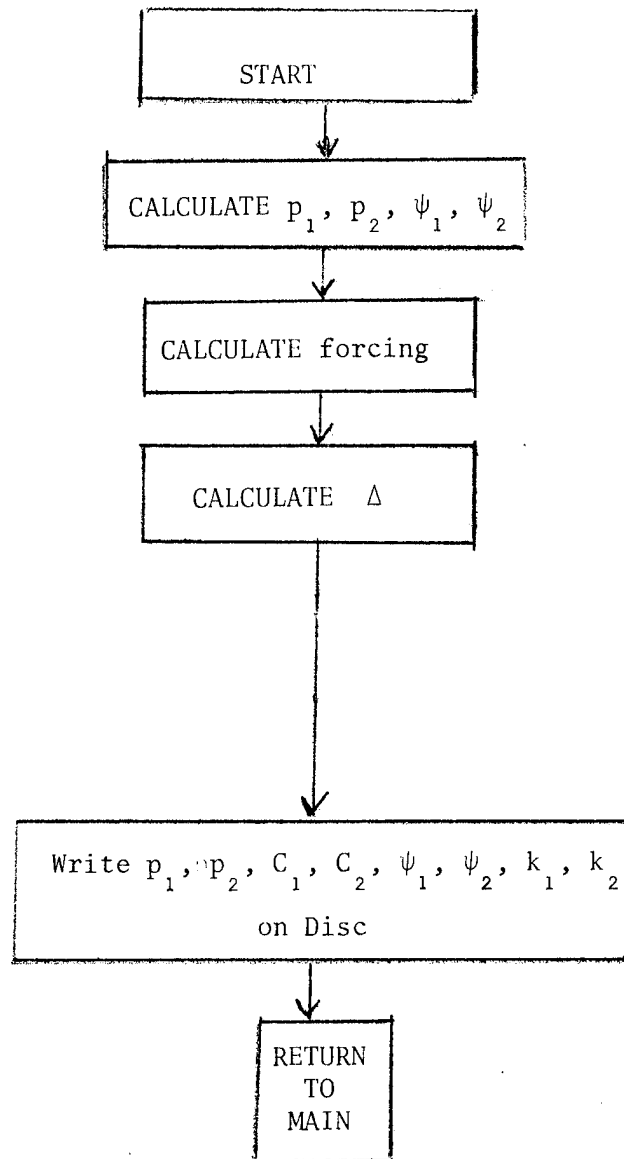
APPENDIX B

A FLOW CHART OF THE PROGRAM

1. Main Program



3. Subroutine calcul



## APPENDIX C

### NOTES ON COMPUTER PROGRAM

The choice of scales is very essential to the success of the present computational method. It is noticed that the accuracy depends on the number of points employed and the range of frequency spectra covered. Considering limitations of both computer storage and time, a time-space grid of  $32 \times 64$  points was chosen for  $t > 0$ ,  $x_1 > 0$ . Thus, the non-dimensionalization of all equations and quantities are both necessary and important to the obtaining of meaningful data from the limited grid size.

The numerical inaccuracies introduced have several origins:

1. Theoretically, error has been introduced by the neglecting of the outer branch points contributions. It has been shown, for isotropic cases, the contributions of these branch points behaved like  $r^{-2}$  at large distance from a delta function loading at the origin  $r = 0$ . Compared with the  $r^{-\frac{1}{2}}$  decreases of the contribution of the residue. Thus, for small  $r$ , or near the origin, the errors might be significant. An asymptotic form of the behavior of small  $r$  has been deduced for a simple delta loading at origin on an isotropic half-space [ ]. It is shown the error thus introduced is of the order of 5% maximum response in stress.
  2. The aliasing error introduced through the periodizing of the functions.
  3. The round-off error in Laplace inversion along with aliasing error.
- have been discussed in the determination of  $C_0$  (Appendix A). It is

found that error 3 is more serious of the two. Hence in calculations the maximum non-dimensionalized time should be restricted to below 6 or 8 for reasonably good results.

4. Errors due to reflections at the boundary. Since the space grid is finite, it has been observed that whenever a wave hit the boundary of chosen space, a sizable numerical error will start propagating in, as if the wave were reflected from the boundary. The basic reason is due to the periodization of the space  $(x_1)$  domain. In computation, this error should be avoided. To correctly determine the extent of the space  $(x_1)$  domain, a priori recognition of significant wave speed (at which most of the energy travels) is important. Usually an estimation will be sufficient. Then the proper nondimensionalizing constants can be chosen.

# APPENDIX D

## PROGRAM INPUT AND OUTPUT

### A. Method:

FFT algorithm for numerical inversion

### B. Input Data Cards:

Card 1. NTEST, NSTRIP, NP (315)

NTEST = 1 A test program for FFT (2-D) (beam).

0 The present program

NSTRIP = 1 With strip

0 No strip

NP = 1 Calculate:  $u_1$  only

2  $u_1, u_3$

3  $u_1, u_3, t_{33}$

4  $u_1, u_3, t_{33}, t_{11}$

5  $u_1, u_3, t_{33}, t_{11}, t_{13}$

Card 2. CC(I) I = 1,9, RHO, ANGLE (8E10,4)

CC(I=1,9) Material constants of composite, in the order

$C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{12}, C_{13}, C_{23}$ . (psi)

RHO Density of composite ( $g/cm^3$ )

ANGLE (degrees) lay-up angle

Card 3. VEL, DM, E1, ANU, DEN (8E10.4)

VEL - Velocity of incoming particle m/sec

DM - Diameter of impacting object cm

E1 - Youngs Modulus of impacting object (psi)

ANU - Poissons ratio of impacting object

DEN - Density of impacting object (gm/cc)



Card 4. NSTRES, NK3, DX3 (215, F10,3)

NSTRES = 1

NK3: Number of steps in  $x_3$  direction

DX3:  $\Delta x_3 / \ell$ , step size in  $x_3$  direction

Photo elastic fringe order computer map in the x-t plane.

Card 5. RO, W, ES (3E10,4)

RO: density of strip beam ( $\text{g/cm}^3$ )

W: depth of strip beam (cm)

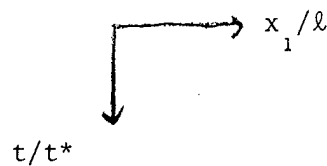
ES: Young's modulus of strip beam (psi)

C. Output:

1. Test problem: Appendix III

2. Values of displacements    1 :  $u_1$   
                                     2 :  $u_3$   
                                     (stress) 3 :  $t_{33}$   
   4 :  $t_{13}$   
   5 :  $t_{11}$

3. Relative magnitude computer maps of displacements and stresses.



```

C***** PROGRAM TO CALCULATE STRESSES DUE TO EDGE IMPACT OF A PLATE*
C
C      THIS PROGRAM CALCULATES THE ELASTIC RESPONSE OF AN
C      ANISOTROPIC PLATE TO AN IN-PLANE EDGE IMPACT FORCE ON  $X_3=0.0$ 
C      WHEN  $NSTRIP=1$ , THE PROGRAM PLACES AN IMPACT PROTECTION STRIP
C      ,OR ELASTIC BEAM ON THE EDGE. THE IMPACT FORCE IS A HALF SINE
C      FUNCTION IN TIME AND IS NON ZERO FOR  $0 < T < T_0$ , (MICROSEC). THE
C      FORCE IS DISTRIBUTED ALONG THE EDGE AS  $P_0(1-X_1^{**2})$ , WHERE
C       $X_1$  IS NORMALIZED BY THE HALF WIDTH OF THE IMPACT CONTACT
C      LENGTH.
C      THE METHOD EMPLOYS A FOURIER TRANSFORM IN THE EDGE DIRECTION
C       $X_1$ , AND A LAPLACE TRANSFORM IN THE TIME DIMENSION. THE TRANSFORM
C      OF THE FORCING FUNCTION IS GIVEN IN THE SUBROUTINE CALCUL AND
C      THE SOLUTION IS OBTAINED USING A 2-DIMENSIONAL FAST FOURIER
C      INVERSION ROUTINE CALLED 'FOURT'.
C      THE OUTPUT FOR A GIVEN DEPTH  $X_3$  CONSISTS OF DISPLACEMENTS
C       $U_1, U_3$ , AND STRESSES  $T_{33}, T_{11}, T_{13}$ , IN THE  $X_1$ -TIME PLANE .
C      THE STRESSES ARE NORMALIZED BY THE ELASTIC CONSTANT  $C(6,6)$ .
C
C                                     IMPACT FORCE
C                                     I
C                                     I
C                                     I
C                                     I
C                                     V I V
C                                     V I V
C                                     VIV
C                                     V
C
C      STRIP >>> * * * * * <<< STRIP
C
C      C ----->  $X_1$ 
C      C |
C      C COMPOSITE C |
C      C |
C      C HALF SPACE C |
C      C V
C      C  $X_3$ 
C
C      INPUT DATA
C      NN(1)--2.0* MAX  $X_1$  DISTANCE, NN(2)--MAX NO. OF TIME UNITS
C      NSTRIP=0, NO STRIP, ...NSTRIP=1, WITH STRIP
C      NP=1, --, 5, CALCULATES  $U_1, U_3, T_{33}, T_{11}, T_{13}$ , IN THAT ORDER
C      NP=6, CALCULATES DISPL. OF A BEAM ON AN ELASTIC FOUNDATION
C      CC(9), ELASTIC CONSTANTS OF PLATE IN THE ORDER
C      C11, C22, C33, C44, C55, C66, C23, C13, C12, IN PSI
C      RHO, DENSITY OF PLATE IN UNITS GM/CC
C      ANGLE--LAYUP ANGLE OF COMPOSITE PLATE, DEG., FOR INFO ONLY
C      VEL, VELOCITY OF INCOMING OBJECT METERS/SEC
C      DM, DIAMETER OF IMPACTING OBJECT -CM.
C      E1, ANU, YOUNG'S MODULUS AND POISSON'S RATIO FOR IMPACTING BODY
C      DEN, DENSITY OF IMPACTING BODY
C      NSTRESS=1
C      NK3, NO.OF DEPTHS  $X_3$ , (FOR  $NK3=1, X_3=0$ )
C      DX3, INCREMENT IN DEPTH  $X_3$  (NORMALIZED BY  $A_0$ )
C      RO, DENSITY OF PROTECTIVE STRIP GM/CC

```

\* This program has two extra cards to override the Hertz contact time and contact length calculation. Remove cards #51, 52.

```

C      W, THICKNESS TO WIDTH RATIO OF BEAM
C      ES, YOUNG'S MODULUS FOR BEAM
C      WL--FOURIER WAVELENGTH (CM) OF THE ORDER OF A0 OR LESS
C      WT--FOURIER WAVE PERIOD (SEC) OF THE ORDER OF T0 OR LESS
C      CHOICE OF WL,WT DETERMINES DX,DT--DX=WL/2,DT=WT/2
C      FG, TRANSFORM OF NORMALIZED FORCING FUNCTION F(X1)*G(T)
C      THIS IS PROVIDED IN PROGRAM BUT CAN BE CHANGED BY THE USER
C
C      OUTPUT DATA
C      TC,TC CONTACT TIME (SEC,1.E-6 SEC ) (FROM HERTZ THEORY)
C      A0,A, HALF THE IMPACT CONTACT LENGTH ,CM
C      F0, MAX IMPACT FORCE FROM HERTZ THEORY ,NEWTONS
C      E, WAVE SPEED IN PLATE SQRT(C66/RHO)
C      OR WAVE SPEED IND BEAM STRIP,SQRT(E1/DEN) UNITS CM/SEC
C      CL=SQRT(C11/RHO) LONGITUDINAL WAVE SPEED ALONG EDGE IN PLATE
C      CS=SQRT(C55/RHO), SHEAR SPEED ALONG EDGE OF PLATE,CM/SEC
C      CR, RAYLEIGH WAVE SPEED ALONG FREE EDGE OF PLATE
C
C      DX,DT SPACE TIME INCREMENTS IN X1-T SPACE UNITS--CM AND SEC
C      DATA(I,J) ,NORMALIZED TRANSFORM OF ONE OF DISPL. OR STRESSES
C      --BEFORE CALL FOURT,AFTER CALL FOURT DATA IS A 2 DIM MATRIX OF
C      DISPL. OR STRESSES IN X1-T SPACE,DEPENDING ON VALUE OF K
C      IN THE LOOP 'DO 4 K=1,NP'
C      U1,U3,NORMALIZED DISPL. IN PLANE OF PLATE(E.G. U1/A0)
C      T33,T11,T13, NORM. STRESSES (E.G. T33/C66)
C      --NOTE-- T33 ON X3=0 SHOULD REPRODUCE THE FORCING FUNCTION
C      WHEN THERE IS NO STRIP
C      --NOTE-- AS A CHECK T13=0 ON X3=0 WHEN THERE IS NO STRIP
C      THE FRINGE ORDER MAP PLOTS THE DIFF IN PRINCIPAL STRESSES AND
C      MAY BE USED TO COMPARE WITH PHOTOELASTIC EXPERIMENTS OR TO
C      LOOK FOR POINTS OF MAX IN PLANE SHEAR STRESSES
C      *****
C      THIS PROGRAM HAS BEEN WRITTEN BY F.MOON AND C-K KANG UNDER
C      A GRANT TO PRINCETON UNIVERSITY FROM THE NASA LEWIS RESEARCH
C      LAB.
C      *****
C      --NOTE TO THE USER-- IN THE OUTPUT MAPS OF STRESSES OR DISPL.,
C      YOU WILL NOTICE BANDS OF SIMILAR NUMBERS RUNNING FROM THE
C      UPPER LEFT CORNER TO THE LOWER RIGHTCORNER -- THESE ARE WAVES
C      WHICH EMINATE FROM THE IMPACT POINT--HOWEVER- WAVES RUNNING
C      FROM RIGHT UPPER TO LEFT LOWER CORNER ARE SPURIOUS DUE TO THE
C      DISCRETENESS OF THE NUMERICAL FOURIER INVERSION PROGRAM
C      --ALSO DATA FOR TIMES NEAR TMAX AT THE BOTTOM OF THE MAPS ARE
C      USUALLY SPURIOUS AND SHOULD NOT BE USED
C

```

```

0001 COMMON /MMC/ D,DK1
0002 COMMON /MC/DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRIP,A0
0003 DIMENSION DATA(128,32),MM(40),CC(9)
0004 DIMENSION NN(2)
0005 DIMENSION RDATA(40)
0006 DIMENSION CIDATA(40)
0007 DIMENSION FRNGE(64,32)
0008 DIMENSION T11(64,32),T33(64,32),T13(64,32)
0009 COMPLEX DATA,S
0010 COMPLEX P1,P2,S1,S2,C1,C2,B,C,D,SI
0011 COMPLEX DK2,FG,SLAP,DK1

```

```

0012      CCMPLX*16 BS
0013      NN(1)=128
0014      NN(2)=32
C***      READ IN AND WRITE OUT DATA AND PARAMETERS
C***      RHO,A0,T0 MUST BE IN C.G.S. UNITS, P0 MUST BE CONSISTENT WITH CC
0015      CALL INDUMP
0016      READ (5,102) NTEST,NSTRIP,NP
0017      READ (5,100) CC,RHO,ANGLE
0018      WRITE(6,490) CC
0019      WRITE(6,491) RHO,ANGLE
0020      READ(5,100) VEL,DM,E1,ANU,DEN
0021      P0=CC(6)
0022      C11= CC(1)-CC(7)**2/CC(2)
0023      C33= CC(3)-CC(9)**2/CC(2)
0024      C13= CC(8)-CC(7)*CC(9)/CC(2)
0025      C55= CC(5)
0026      PI= 3.14159265
0027      RHO= RHO/6.895*1.E-4
0028      READ (5,101) NSTRES,NK3,DX3
0029      WRITE (6,504) NSTRES,NK3,DX3
0030      SI= CMLPX(0.,1.)
C*****      CALCULATE THE IMPACT CONTACT TIME ,RADIUS,AND PRESSURE
C*****      BASED ON HERTZ CONTACT THEORY
0031      R=DM/2.0
0032      R=R*1.E-2
0033      E1=E1*6895.0
0034      E2=CC(2)*6895.0
0035      DEN=DEN*1.0E3
0036      AMASS=4./3.*PI*R**3*DEN
0037      AK2=4./3.*SQRT(R)*E1/((1.0-ANU*ANU)+E1/E2)
0038      ALF=5./4.*AMASS*VEL*VEL/AK2
0039      ALF= (ALF)**0.4
0040      TC=2.943*ALF/VEL
0041      P0=1.14*AMASS*VEL*VEL/ALF
0042      A=SQRT(ALF*R)
0043      A=1.0E2*A
0044      TC=TC*1.0E6
0045      WRITE(6,710) VEL,TC,A,P0
C*****
C***      CALCULATE THE NON-DIMENSIONAL PARAMETERS
0046      EE= CC(6)
0047      E= SQRT(EE/RHO)
0048      RE= RHO
C      DEFINE TRANSFORM SPACE AND DISTANCE-TIME SPACE
C***      UNIT DISTANCE  --CM.
0049      A0=A
0050      T0=TC*1.E-6
C      TEST CASE T0== 35E-6, A0=1 CM
0051      A0=1.0
0052      T0=35.E-6
0053      AL=A0
C***      UNIT TIME--SEC.
0054      TE=AL/E
C***      SMALLEST WAVELENGTHS
0055      WL=A0/1.5
0056      WT=TC/10.0

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0057 C*** DIMENSIONS OF TRANSFORM SPACE
0058     XK=2.0*PI*AL/WL
0059     TK=2.0*PI*TE/WT
0060 C*** NCNDIMENSIONALIZE CONSTANTS
0061     A0=A0/AL
0062     T0=T0/TE
0063     DT=WT/2.0
0064     DX=WL/2.0
0065     WRITE(6,621) WT,WL,TK,XK
0066 C*****
0067 C      CALCULATE RAYLEIGH WAVE SPEED
0068     X1=0.0
0069     X2=C55
0070     N=0
0071     DO 90 I=1,100
0072     N=N+1
0073     X=(X1+X2)/2.0
0074     D1=(C55-X)/(C11-X)
0075     D1=SQRT(D1)
0076     D2=(C13)**2-C33*(C11-X)
0077     D3=C55*C33
0078     D3=SQRT(D3)
0079     F=D1*D2/D3
0080     F=X+F
0081     IF(F) 81,82,83
0082 81 X1=X
0083     GO TO 92
0084 82 GO TO 91
0085 83 X2=X
0086 92 CONTINUE
0087     F1=F/C55
0088     DIFF=ABS(F1)-1.0E-4
0089     IF(DIFF) 91,91,90
0090 90 CONTINUE
0091 91 CONTINUE
0092     WRITE(6,701) C11,C33,C55,C13
0093 701 FORMAT(//,10X,6H C11 =,E12.4,6H C33 =,E12.4,6H C55 =,E12.4,
0094     16H C13 =,E12.4,/)
0095     CL=C11/RHO
0096     CL=SQRT(CL)
0097     CS=C55/RHO
0098     CS=SQRT(CS)
0099     CR=X/RHO
0100     CR=SQRT(CR)
0101     RDS=CR/CS
0102     WRITE(6,702) CL,CS,CR,RDS,N
0103 702 FORMAT(//,10X,12H LONG. SPEED =,E12.4,13H SHEAR SPEED =,E12.4
0104     1 //,10X,16H RAYLEIGH SPEED =,E12.4,8H CR/CS =,F10.5,5X,15)
0105 C*****
0106 C*** CONSTANTS FOR STRIP CASE
0107     READ(5,100) RO,W,ES
0108     WRITE(6,521) RO,W,ES
0109     RO= RO/6.895*1.E-4
0110     IF (RO) 51,51,50
0111 50 CONTINUE
0112     E= SQRT(ES/RO)

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0105      WRITE (6,510) E
0106      51 CONTINUE
0107      RO= RO/RE
0108      ES= ES/EE
0109      W= W/AO/AL
C*****
0110      205 CONTINUE
0111      C11= C11/EE
0112      C13= C13/EE
0113      C33= C33/EE
0114      C55= C55/EE
0115      P0= P0/EE
0116      RHO= RHO/RE
0117      FG1= P0*T0
0118      206 CONTINUE
0119      N= NN(1)
0120      M= NN(2)
C***  CALCULATE THE LAPLACE INVERSION PARAMETER
0121      WT=WT/TE
0122      RH= P0*WT*1.E6
0123      CH= 2./3./M/WT*ALOG(RH)
0124      CLAP= CH
C***  CALCULATE THE SECOND ORDER BRANCH POINTS
0125      A= RHO*(C33-C55)**2*RHO
0126      E= -2.*RHO*((C33+C55)*(C13*C13+2.*C13*C55-C11*C33)+2.*C33*C55
1*(C11+C55))
0127      F= (C13*C13+2.*C13*C55-C11*C33)**2-4.*C11*C33*C55*C55
0128      BS= 1.D0*E*E-1.D0*4.*A*F
0129      D= CDSQRT(BS)
0130      P1= .5/A*(-E+D)
0131      P2= F/P1/A
0132      P1= CSQRT(P1)
0133      P2= CSQRT(P2)
0134      WRITE (6,509)
0135      WRITE (6,506) P1,P2
0136      204 WRITE (6,508) CLAP
0137      N2= N/2+1
0138      M2= M/2+1
0139      IF (NTEST.EQ.1) GO TO 211
0140      211 IF (NSTRIPEQ.1) WRITE (6,514)
C***  GENERATE THE TRANSFORMED EXPRESSIONS
0141      REWIND 2
0142      DO 1 I= 1,N
0143      DK1= 2.*XK/N*(I-.5)-XK
0144      DO 1 J= 1,M
0145      DK2F=2.*TK/M*(J-.5)-TK
0146      SLAP= CMPLX(CLAP,DK2F)
0147      DK2= -SI*SLAP
0148      IF (NTEST.NE.1) GO TO 201
0149      FG= PI*(1.+CEXP(-SLAP))/(SLAP*SLAP+PI*PI)*
1CSIN(DK1)/DK1*(1.+DK1*DK1/(PI*PI-DK1*DK1))
C***  TEST FOR A STRING
0150      DATA(I,J)=-FG/(SLAP*SLAP+DK1*DK1)
0151      GO TO 1
0152      201 CALL CALCUL(0)
0153      1 CONTINUE

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FORTTRAN IV G LEVEL 21

MAIN

DATE = 74186

11/28/2

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0154      IF (NTEST.NE.1) GO TO 202
0155      WRITE (6,507)
0156      CALL FOURT (DATA,NN,2,1,1,0)
0157      DO 15 KI= 1,N2
0158      B= -SI*PI/N*(N-1)*(KI-1)
0159      DO 15 KJ= 1,M
0160      C= -SI*PI/M*(M-1)*(KJ-1)
0161      T= PI/TK*(KJ-1)
0162      DATA (KI,KJ)= DATA (KI,KJ)*XK*TK/PI/PI/N/M*CEXP(B)*CEXP(C)
0163      15 DATA (KI,KJ)= DATA (KI,KJ)*EXP(CLAP*T)
0164      WRITE (6,506) ((DATA(I,J), J= 1,M), I= 1,N2)
0165      GO TO 203
C*****
C***  MAIN LOOP FOR DIFFERENT DEPTHS X3
0166      202 DO 6 K3= 1,NK3
0167      X3= (K3-1)*DX3
C***  LOOP FOR CALCULATING DISPLACEMENTS AND STRESSES AT A GIVEN DEPTH
0168      DO 4 K= 1,NP
0169      WRITE (6,680)
0170      WRITE (6,501) X3
0171      REWIND 2
0172      DO 7 I= 1,N
0173      DO 7 J= 1,M
0174      READ (2) P1,P2,C1,C2,S1,S2,DK1,DK2
0175      B= CEXP(-X3*P1)
0176      C= CEXP(-X3*P2)
C***  DISPLACEMENTS
0177      IF (K.EQ.1) DATA (I,J)= C1*B+C2*C
0178      IF (K.EQ.2) DATA (I,J)= C1*S1*B+C2*S2*C
C***  STRESSES
0179      IF (K.EQ.3)
0180      1DATA (I,J)= (C13*SI*DK1-C33*P1*S1)*C1*B+(C13*SI*DK1-C33*P2*S2)*C2*C
0181      IF (K.EQ.4)
0182      1DATA (I,J)= (C11*SI*DK1-C13*P1*S1)*C1*B+(C11*SI*DK1-C13*P2*S2)*C2*C
0183      IF (K.EQ.5)
0184      1DATA (I,J)= C55*((-P1+SI*DK1*S1)*C1*B+(-P2+SI*DK1*S2)*C2*C)
0185      IF (K.LT.6) GO TO 7
C***  DISPL. FOR A BEAM ON AN ELASTIC FOUNDATION
0186      EF=0.1
C***  FORCING FUNCTION
0187      FG= PI*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI-DK2*DK2*T0*T0)*
0188      14./(DK1*A0)**2*(CSIN(DK1*A0)/DK1-A0*CCOS(DK1*A0))
0189      D=W*DK1*DK1/12.0*(ES*W*W*DK1*DK1-RO*W*W*DK2*DK2)-RO*W*DK2*DK2
0190      1 +EF
0191      IF (K.EQ.6)
0192      1DATA (I,J)=FG/D
0193      7 CONTINUE
0194      CALL FOURI (DATA,NN,2,1,1,0)
0195      DO 3 KI= 1,N2
0196      B= -SI*PI/N*(N-1)*(KI-1)
0197      DO 3 KJ= 1,M
0198      C= -SI*PI/M*(M-1)*(KJ-1)
0199      T= PI/TK*(KJ-1)
0200      DATA (KI,KJ)= DATA (KI,KJ)*XK*TK/PI/PI/M/N*CEXP(B)*CEXP(C)
0201      E= CLAP*T
0202      3 DATA (KI,KJ)= DATA (KI,KJ)*EXP(E)

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ORTTRAN IV G LEVEL 21

MAIN

DATE = 74186

11/28/2

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C*** CALCULATE THE PHOTOELASTIC FRINGE ORDER
0197 DO 290 I=1,40
0198 DO 290 J=1,32
0199 IF (K.EQ.3) T33(I,J)=REAL(DATA(I,J))
0200 IF (K.EQ.4) T11(I,J)=REAL(DATA(I,J))
0201 IF (K.EQ.5) T13(I,J)=REAL(DATA(I,J))
0202 IF (K.EQ.5) GO TO 280
0203 GO TO 290
0204 280 CONTINUE
0205 FNGE=(T11(I,J)-T33(I,J))**2+4.0*T13(I,J)*T13(I,J)
0206 FRNGE(I,J)=SQRT(FNGE)
0207 290 CONTINUE
0208 214 WRITE (6,511) K
C*** FIND THE MAXIMUM VALUE
0209 RS= 1.E-3
0210 DO 14 I=1,40
0211 DO 14 J= 1,32
0212 S= DATA(I,J)
0213 TP= REAL(S)/RS
0214 IF (ABS(TP).LT.1.) GO TO 14
0215 RS= REAL(S)
0216 14 CONTINUE
0217 WRITE (6,516) RS
C PRINT REAL PART OF DISPL. AND STRESSES
0218 NIJ=N/4
0219 NIJ=10
0220 DO 310 I=1,NIJ
0221 I1=I-1
0222 WRITE(6,600) I1
0223 DO 300 J=1,32
0224 SR=REAL(DATA(I,J))/RS
0225 SIMG=AIMAG(DATA(I,J))/RS
0226 RDATA(J)=SR
0227 CIDATA(J)=SIMG
0228 300 CONTINUE
0229 WRITE(6,620) (RDATA(L),L=1,M)
0230 WRITE(6,650)
0231 WRITE(6,620) (CIDATA(L),L=1,11)
0232 310 CONTINUE
0233 WRITE(6,680)
0234 IF (K.EQ.1) WRITE(6,630)
0235 IF (K.EQ.2) WRITE(6,631)
0236 IF (K.EQ.3) WRITE(6,632)
0237 IF (K.EQ.4) WRITE(6,633)
0238 IF (K.EQ.5) WRITE(6,634)
0239 IF (K.EQ.6) WRITE(6,635)
0240 WRITE(6,640) DX,DT
C*** PLOT THE RELATIVE VALUES
0241 DO 12 J= 1,M
0242 DO 13 I=1,40
0243 S= DATA(I,J)
0244 13 MM(I)= REAL(S)/RS*100
0245 WRITE (6,515) MM
0246 12 CCNTINUE
C*** PLOT A MAP OF PHOTOELASTIC FRINGE ORDER
0247 IF (K.LT.5) GO TO 4

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0248      RS=1.E-3
0249      DO 312 I=1,20
0250      DO 312 J=1,20
0251      SS=FRNGE(I,J)
0252      TP=SS/RS
0253      IF (ABS(TP).LT.1.) GO TO 312
0254      RS=SS
0255      312 CONTINUE
0256      WRITE(6,680)
0257      WRITE(6,516) RS
0258      WRITE(6,690)
0259      DO 320 J=1,32
0260      DO 315 I=1,40
0261      SS=FRNGE(I,J)
0262      315 MM(I)=SS/RS*100
0263      WRITE(6,515) MM
0264      320 CONTINUE
0265      4 CONTINUE
0266      6 CONTINUE
0267      100 FORMAT (8E10.4)
0268      101 FORMAT (2I5,3F10.3)
0269      102 FORMAT (3I5)
0270      490 FORMAT(5X,55H ELASTIC CONSTANTS C11,C22,C33,C44,C55,C66,C12,C13,C3
12,/,9E12.4,/)
0271      491 FORMAT(5X,25H DENSITY OF PLATE GM/CC--,F10.4,5X,20H FIBER LAYUP AN
1GLE--,F10.4)
0272      500 FORMAT (2F12.4,4E15.4)
0273      501 FORMAT (5H X3= F10.4)
0274      502 FORMAT (10H STRESSES I3/(8E15.7))
0275      503 FORMAT (5X5H DATA, 15X10HCONSTANTS ,20X10HPARAMETERS /8E15.7)
0276      504 FORMAT (5X8HNSTRESS= I3,5X4HNX3= I4,5X4HDX3= F10.4,/)
0277      505 FORMAT (14H SIMPLE POLES )
0278      506 FORMAT (8E15.7)
0279      507 FORMAT (15H THIS IS A TEST )
0280      508 FORMAT (30H LAPLACE INVERSION PARAMETER= F12.4)
0281      509 FORMAT (28H SECCND ORDER BRANCH POINTS )
0282      510 FORMAT(5X,27H LONG.WAVE SPEED IN BEAM = ,F12.3,7H CM/SEC)
0283      511 FORMAT (14H DISPLACEMENTS I4)
0284      512 FORMAT (24H RAYLEIGH SPEED CS/CR*I )
0285      513 FORMAT (8F15.8)
0286      514 FORMAT (/50X 10HWITH STRIP)
0287      515 FORMAT(2X,40I3)
0288      516 FORMAT (16H MAXIMUM VALUE= E15.7)
0289      520 FORMAT(/,5X,23H IMPACT PRESSURE(PSI)= ,E12.4,5X,25H 1/2 CONTACT LE
1NGTH(CM)= ,F12.4,5X,23H IMPACT DURATION(SEC)= ,E12.4,/)
0290      521 FORMAT(/,5X,18H DENSITY OF BEAM= ,F12.4,5X,17H NORM.THICKNESS= ,
1F12.4,5X,15H BEAM MODULUS= ,E12.4)
0291      600 FORMAT(/, 20X,18HNORMALIZED DIST.= ,I4)
0292      620 FORMAT(11F10.5)
0293      621 FORMAT(5X,20H WAVELEN (TIME-SEC)= ,E12.4,5X,20H WAVELEN(DIST-CM )=
1,F12.4,/,5X,23H MAX FREQ NO(NON DIM)= ,F12.4,5X,23H MAX WAVE NO(NC
2N DIM)= ,F12.4,/)
0294      630 FORMAT(30X,16H DISPLACEMENT U1,/)
0295      631 FORMAT(30X,16H DISPLACEMENT U3,/)
0296      632 FORMAT(30X,11H STRESS T33,/)
0297      633 FORMAT(30X,11H STRESS T11,/)

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ORTRAN IV G LEVEL 21

MAIN

DATE = 74186

11/28/2

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0298      634 FORMAT(30X,11H STRESS T13,/)
0299      635 FORMAT(30X,41H TEST-DISPL.OF BEAM ON ELASTIC FOUNDATION,/)
0300      640 FORMAT(20X,14H-->> X1, DX= ,F12.4,/,20X,2H 1,/,20X,2H 1,/,20X,
      1 2H 1,/,20X,2H V,/,20X,2H V,/,20X,9H TIME,DT= ,E12.4,/)
0301      650 FORMAT(10H IMAG PART)
0302      680 FORMAT(1H1)
0303      690 FORMAT(/,20X,17H FRINGE ORDER MAP,/,20X,31H SQRT((T11-T33)**2+4.0*
      1T13*T13))
0304      710 FORMAT( 5X,6H VEL= ,F12.4,5X,5H T0= ,F12.4,5X,4H A= ,F12.4,5X,
      15H P0= ,F12.4,/)
0305      203 STOP
0306      END
```

C

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0001      SUBROUTINE CALCUL(NPOLE)
0002      COMMON /MMC/ D,DK1
0003      COMMON /MC/DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRIP,A0
0004      CCMPLX B1,B2
0005      CCMPLX G0,G01,H0,H01,G10,G11,H10,H11
0006      COMPLEX G,H,G1,H1
0007      COMPLEX DK1,DK2,P1,P2,S1,S2,C1,C2,SI,B,C,D,FG
0008      CCMPLX*16 BS
0009      PI= 3.14159265
0010      SI= CMPLX(0.,1.)
0011      A= C33*C55
0012      B= C33*(RHO*DK2*DK2-C11*DK1*DK1)+C55*(RHO*DK2*DK2-C55*DK1*DK1) +
1DK1*DK1*(C13+C55)**2
0013      C= (RHO*DK2*DK2-C11*DK1*DK1)*(RHO*DK2*DK2-C55*DK1*DK1)
0014      BS= 1.D0*B*B-1.D0*4.*A*C
0015      D= CDSQRT(BS)
0016      P1= .5/A*(-B+D)
0017      P2= C/P1/A
0018      P1= CSQRT(P1)
0019      IF (REAL(P1).LT.0.0.AND.NPOLE.NE.2) P1= -P1
0020      P2= CSQRT(P2)
0021      IF (REAL(P2).LT.0.0.AND.NPOLE.NE.2) P2= -P2
0022      S1= -SI/DK1 /P1/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P1*P1)
0023      S2= -SI/DK1 /P2/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P2*P2)
0024      IF (NPOLE.EQ.2) GO TO 201
C***  FORCING FUNCTION
0025      FG= PI*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI-DK2*DK2*T0*T0)*
14./ (DK1*A0)**2*(CSIN(DK1*A0)/DK1-A0*CCOS(DK1*A0))
0026      201 IF (NSTRIP.EQ.1) GO TO 202
0027      D= (SI*DK1*S1-P1)*(SI*DK1*C13-P2*C33*S2)-(SI*DK1*S2-P2)*(SI*DK1*
1C13-P1*C33*S1)
0028      IF (NPOLE.EQ.2) RETURN
0029      206 C1= FG/D*(SI*DK1*S2-P2)
0030      C2= FG/D*(P1-SI*DK1*S1)
0031      GO TO 203
0032      202 CONTINUE
C***  MATRIX FOR STRIP PROBLEM
0033      B1=WWW*(ES*DK1*DK1-RO*DK2*DK2)*DK1*DK1/12.0-RO*DK2*DK2
0034      B1=W*B1
0035      B2=-W*(ES*DK1*DK1-RO*DK2*DK2)
0036      G10= C55*(SI*DK1*S1-P1)
0037      H10= C55*(SI*DK1*S2-P2)
0038      G11=B2*(1.0+SI*DK1*S1*W/2)
0039      H11=B2*(1.0+SI*DK1*S2*W/2)
0040      G0=- (SI*DK1*C13-P1*S1*C33)
0041      H0=- (SI*DK1*C13-P2*S2*C33)
0042      G01=-SI*DK1*W*G10/2.0 + B1*S1
0043      H01=-SI*DK1*W*H10/2.0 + B1*S2
0044      H1=H10+H11
0045      G1=G10+G11
0046      G=G0+G01
0047      H=H0+H01
0048      D=G10*H0-G0*H10 + (H0*G11+H01*G10-G0*H11-G01*H10)+(H01*G11-H11*G01
1)
0049      IF (NPOLE.EQ.2) RETURN

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FORTTRAN IV G LEVEL 21

CALCUL

DATE = 74186

11/28/2

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0050      C1= -FG/D*H1
0051      C2= FG/D*G1
0052      203 WRITE (2) P1,P2,C1,C2,S1,S2,DK1,DK2
0053      RETURN
0054      END
```

ELASTIC CONSTANTS C11,C22,C33,C44,C55,C66,C12,C13,C32,  
0.2456E 08 0.1170E 07 0.1374E 07 0.3552E 06 0.2310E 07 0.3552E 06 0.4000E 06 0.1986E 07 0.4558E 06

DENSITY OF PLATE GM/CC-- 1.4400 FIBER LAYUP ANGLE-- 15.0000  
NSTRESS= 1 NX3= 1 DX3= 1.0000

VEL= 300.0000 T0= 87.3169\* A= 1.3342\* F0= 1042938.7500

WAVELEN(TIME-SEC)= 0.3500E-05 WAVELEN(DIST-CM )= 0.6667  
MAX FREQ NO(NON DIM)= 13.7654 MAX WAVE NO(NON DIM)= 9.4248

C11 = 0.2442E 08 C33 = 0.1196E 07 C55 = 0.2310E 07 C13 = 0.1830E 07

LCNG.SPEED = 0.1081E 07SHEAR SPEED = 0.3326E 06

RAYLEIGH SPEED = 0.3020E 06 CR/CS = 0.90800 12

DENSITY OF BEAM= 2.7000 NORM.THICKNESS= 0.5000 BEAM MODULUS= 0.1124E 08  
LCNG.WAVE SPEED IN BEAM = 535757.750 CM/SEC  
SECOND ORDER BRANCH POINTS  
0.1997043E 02 0.0 0.2266004E 01  
LAPLACE INVERSION PARAMETER= 0.5948

WITH STRIP

\* This program has test cards (#51, 52) to override the Hertz calculation, T0=3510<sup>-6</sup> s A=1 cm.



X3= 0.0

DISPLACEMENTS 3

MAXIMUM VALUE= -0.7104896E 00

NORMALIZED DIST.= 0													
0.00050	0.03979	0.14142	0.27084	0.41184	0.54907	0.67705	0.78714	0.87790	0.94377	0.98577			
1.00000	0.98948	0.95134	0.89048	0.80461	0.70045	0.57593	0.43953	0.28894	0.13502	0.01142			
-0.04267	-0.07263	-0.07276	-0.07738	-0.06042	-0.06357	-0.03818	-0.05201	-0.01223	-0.05377				
IMAG PART													
0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00001	0.00002	0.00003			
NORMALIZED DIST.= 1													
0.00052	0.03504	0.12364	0.23925	0.36748	0.49373	0.61244	0.71537	0.80083	0.86362	0.90441			
0.91968	0.91198	0.87880	0.82442	0.74692	0.65225	0.53870	0.41383	0.27579	0.13423	0.01894			
-0.03628	-0.06768	-0.07048	-0.07543	-0.06060	-0.06261	-0.03975	-0.05059	-0.01542	-0.04968				
IMAG PART													
0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00001	0.00002	0.00003			
NORMALIZED DIST.= 2													
0.00046	0.02177	0.07726	0.15621	0.24997	0.34647	0.44004	0.52356	0.59467	0.64909	0.68662			
0.70461	0.70446	0.68451	0.64749	0.59235	0.52308	0.43886	0.34489	0.24037	0.13188	0.03829			
-0.01809	-0.05320	-0.06323	-0.06962	-0.06054	-0.06012	-0.04359	-0.04736	-0.02317	-0.04079				
IMAG PART													
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002			
NORMALIZED DIST.= 3													
0.00028	0.00689	0.02688	0.06236	0.11326	0.17235	0.23427	0.29336	0.34651	0.39036	0.42373			
0.44479	0.45370	0.44958	0.43352	0.40525	0.36662	0.31768	0.26100	0.19681	0.12822	0.06182			
0.00870	-0.03042	-0.04999	-0.06009	-0.05857	-0.05702	-0.04745	-0.04482	-0.03114	-0.03385				
IMAG PART													
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001			
NORMALIZED DIST.= 4													
0.00030	0.00089	0.00200	0.00749	0.02457	0.05280	0.08844	0.12718	0.16551	0.20047	0.23014			
0.25300	0.26837	0.27564	0.27493	0.26619	0.25006	0.22685	0.19759	0.16280	0.12382	0.08191			
0.03868	-0.00176	-0.03008	-0.04672	-0.05288	-0.05368	-0.04925	-0.04457	-0.03642	-0.03159				
IMAG PART													
0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001			
NORMALIZED DIST.= 5													
0.00032	0.00039	-0.00267	-0.01052	-0.01476	-0.00909	0.00649	0.02920	0.05590	0.08350	0.10982			
0.13298	0.15198	0.16592	0.17462	0.17771	0.17551	0.16799	0.15576	0.13895	0.11848	0.09467			
0.06546	0.02911	-0.00428	-0.02892	-0.04250	-0.04904	-0.04829	-0.04595	-0.03865	-0.03469				
IMAG PART													
0.00000	0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000			

**STRESS T11**

--->> X1, DX=	0.3333
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A

2

TIME, DT= 0.1750E-05

[illegible]



X3= 0.0  
DISPLACEMENTS 4  
MAXIMUM VALUE= -0.1169061E 01

		NORMALIZED DIST.= 0									
0.00258	0.00591	-0.02645	-0.05231	-0.04372	-0.00204	0.06720	0.15474	0.25280	0.35369	0.45160	0.51992
0.54124	0.61882	0.68126	0.72641	0.75286	0.75988	0.74715	0.71530	0.66481	0.59890	0.51992	
0.39153	0.25986	0.15272	0.07511	0.01861	-0.01623	-0.04614	-0.05638	-0.08178	-0.05544		
IMAG PART											
0.00000	0.00000	0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	
		NORMALIZED DIST.= 1									
0.00200	0.00707	-0.00660	-0.01783	-0.00073	0.04631	0.11913	0.20892	0.30838	0.40951	0.50684	
0.59461	0.66947	0.72778	0.76810	0.78838	0.78875	0.76829	0.72861	0.66949	0.59492	0.50860	
0.39149	0.26578	0.16154	0.08150	0.02557	-0.01399	-0.04097	-0.05868	-0.07437	-0.06856		
IMAG PART											
0.00000	0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002	
		NORMALIZED DIST.= 2									
0.00095	0.01745	0.05203	0.08323	0.12706	0.19085	0.27497	0.37173	0.47550	0.57739	0.67297	
0.75508	0.82171	0.86766	0.89339	0.89515	0.87552	0.83186	0.76861	0.68362	0.58354	0.48166	
0.39022	0.28142	0.18630	0.10048	0.04569	-0.00618	-0.02674	-0.06290	-0.05665	-0.09664		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	
		NORMALIZED DIST.= 3									
0.00057	0.03187	0.10966	0.18732	0.26129	0.34028	0.43188	0.53084	0.63438	0.73316	0.82397	
0.89837	0.95550	0.98870	1.00000	0.98414	0.94583	0.88079	0.79607	0.68776	0.56538	0.45448	
0.38626	0.30196	0.22148	0.13213	0.07587	0.01127	-0.00827	-0.06039	-0.04181	-0.11182		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00003	
		NORMALIZED DIST.= 4									
0.00019	0.01996	0.08833	0.16952	0.24197	0.30671	0.37709	0.45380	0.53712	0.61997	0.69932	
0.76750	0.82278	0.85892	0.87669	0.87127	0.84603	0.79712	0.73005	0.64180	0.53914	0.43862	
0.38149	0.32254	0.25902	0.17532	0.11222	0.04213	0.01049	-0.04408	-0.03756	-0.10398		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002	
		NORMALIZED DIST.= 5									
0.00069	0.00496	0.04046	0.09727	0.15250	0.19436	0.23295	0.27765	0.33095	0.39123	0.45436	
0.51583	0.57109	0.61642	0.64875	0.66607	0.66710	0.65118	0.61852	0.56999	0.50604	0.43337	
0.38097	0.34064	0.29211	0.22508	0.15284	0.08553	0.03096	-0.01399	-0.04177	-0.07016		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001	0.00002	



X3= 0.0  
DISPLACEMENTS 5  
MAXIMUM VALUE= -0.4623035E 00

		NORMALIZED DIST.= 0									
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00000	-0.00000
-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00000	-0.00000
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
		NORMALIZED DIST.= 1									
-0.00043	0.02068	0.10175	0.20639	0.30728	0.39631	0.47093	0.52964	0.57264	0.59915	0.61001	0.61001
0.60465	0.58452	0.54925	0.50110	0.43993	0.36875	0.28784	0.20073	0.10784	0.01335	-0.06204	-0.06204
-0.07283	-0.05896	-0.03786	-0.02592	-0.01095	-0.00946	0.00347	-0.00564	0.01473	-0.01847		
IMAG PART											
0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001	0.00001	0.00002
		NORMALIZED DIST.= 2									
-0.00013	0.03502	0.15215	0.31697	0.48494	0.63521	0.76207	0.86189	0.93521	0.98069	1.00000	1.00000
0.99229	0.96012	0.90298	0.82454	0.72472	0.60842	0.47603	0.33347	0.18134	0.02696	-0.09592	-0.09592
-0.12971	-0.11354	-0.07743	-0.05318	-0.02567	-0.01857	0.00226	-0.00719	0.01974	-0.01810		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00002	0.00002	0.00003	0.00003
		NORMALIZED DIST.= 3									
-0.00001	0.02103	0.10073	0.24333	0.41035	0.56703	0.70144	0.80789	0.88641	0.93621	0.95909	0.95909
0.95511	0.92661	0.87394	0.80010	0.70587	0.59526	0.46931	0.33299	0.18781	0.03946	-0.09051	-0.09051
-0.15673	-0.15637	-0.11853	-0.08225	-0.04707	-0.02788	-0.00646	-0.00410	0.01312	-0.00197		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00002	0.00002	0.00003	0.00003
		NORMALIZED DIST.= 4									
-0.00004	-0.00297	0.00997	0.07754	0.19345	0.31982	0.43364	0.52662	0.59598	0.64235	0.66643	0.66643
0.67025	0.65479	0.62217	0.57325	0.51049	0.43492	0.34924	0.25488	0.15515	0.05108	-0.05597	-0.05597
-0.14683	-0.17773	-0.15636	-0.11319	-0.07501	-0.04060	-0.02147	-0.00133	0.00071	0.01812		
IMAG PART											
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002	0.00002
		NORMALIZED DIST.= 5									
0.00030	-0.00548	-0.02595	-0.02357	0.03348	0.12663	0.22238	0.30661	0.37192	0.41857	0.44612	0.44612
0.45759	0.45334	0.43645	0.40685	0.36774	0.31856	0.26281	0.19987	0.13350	0.06310	-0.01262	-0.01262
-0.10517	-0.16989	-0.18071	-0.14489	-0.10571	-0.06167	-0.03831	-0.00752	-0.00720	0.02622		
IMAG PART											
0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001	0.00001	0.00002

MAXIMUM VALUE= 0.1168567E 01

FRINGE ORDER MAP

$$\text{SQRT}(\text{T11-T33}) ** 2 + 4.0 * \text{T13} * \text{T13})$$
[illegible]

JOB 0350 7/5/74 IBM 360-91

EXECUTION TIME 34 SEC.

25 PAGES

CORE 150 K

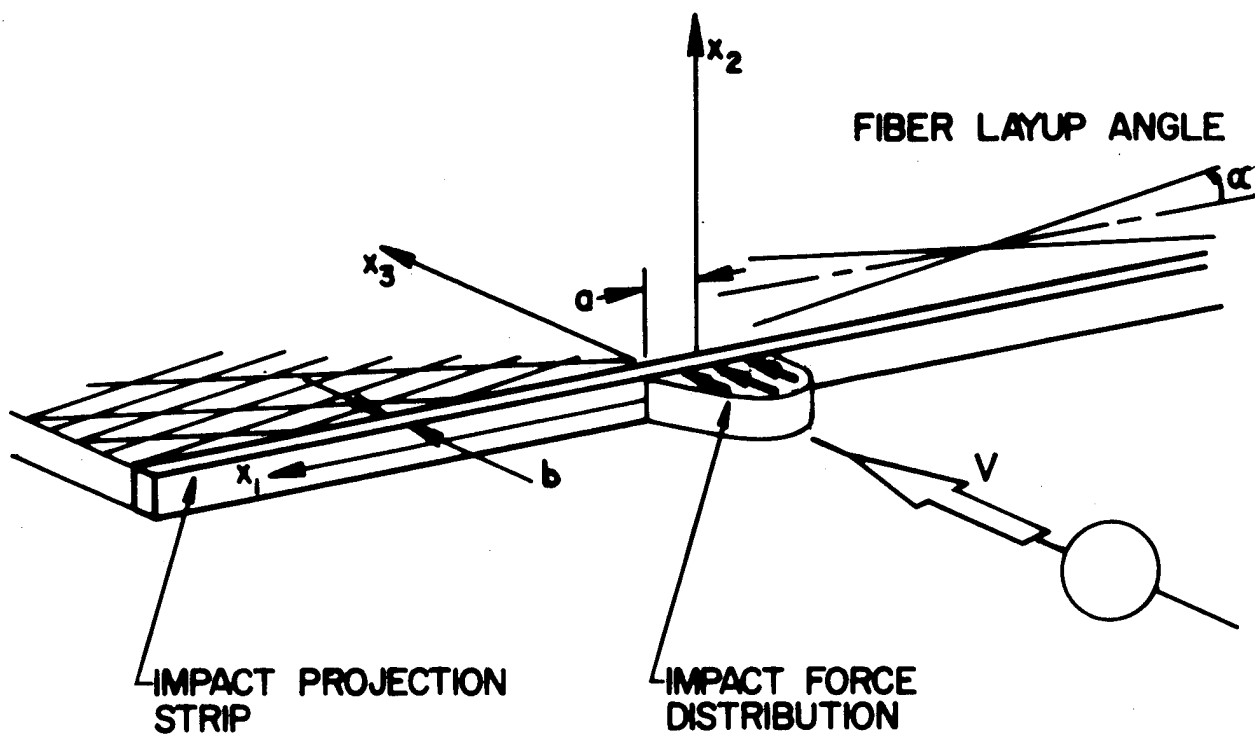


FIGURE 1

GEOMETRY OF IN-PLANE EDGE IMPACT OF  
A COMPOSITE PLATE WITH PROJECTION STRIP

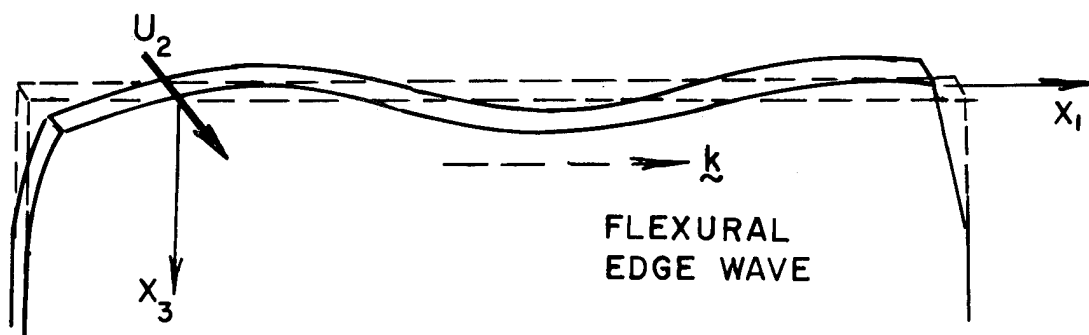
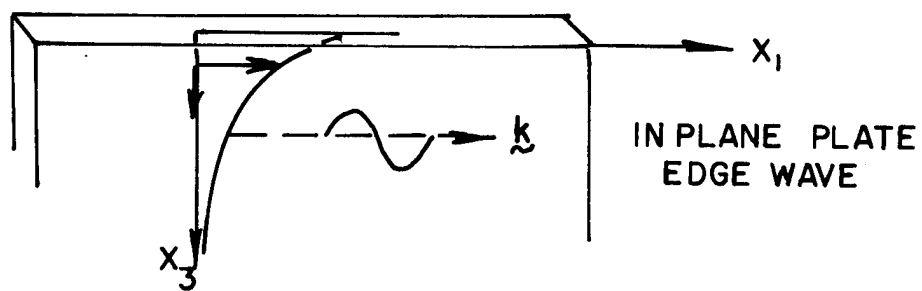
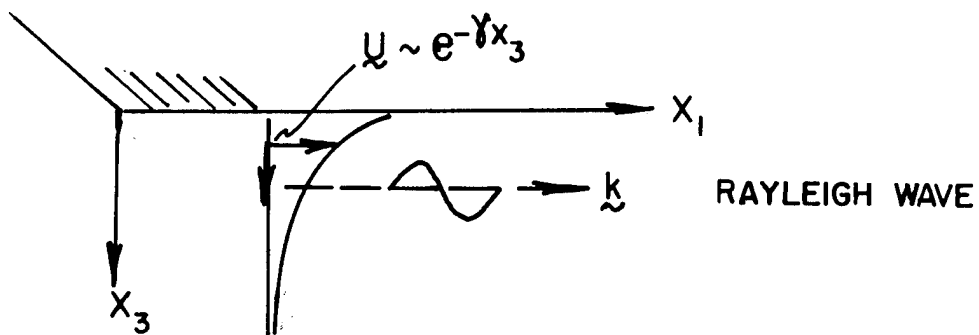


FIGURE 2a

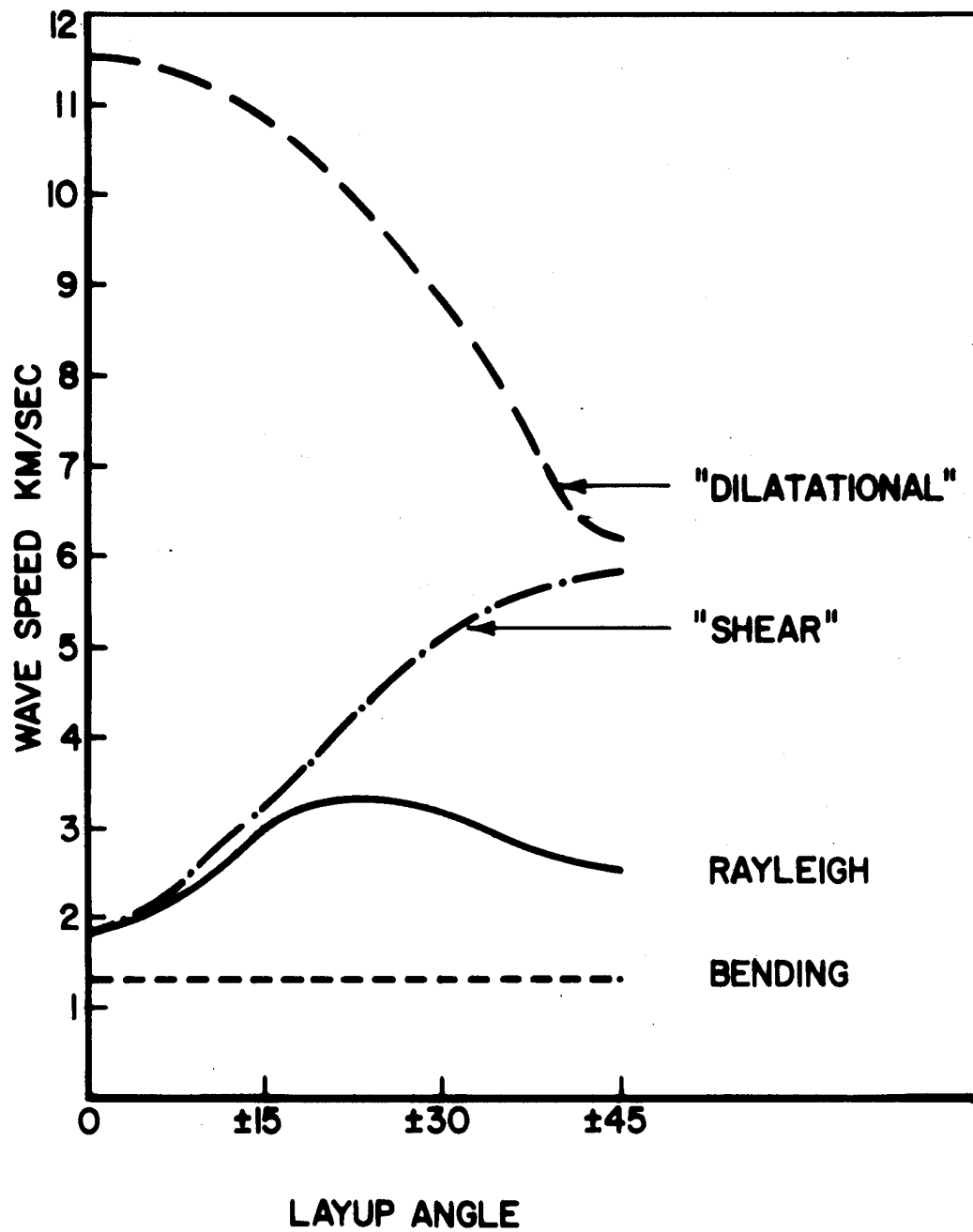


FIGURE 2b

Comparison of In-plane plate edge wave speed with body wave speeds versus fiber layup angle for 55% graphite fiber/epoxy matrix composite.



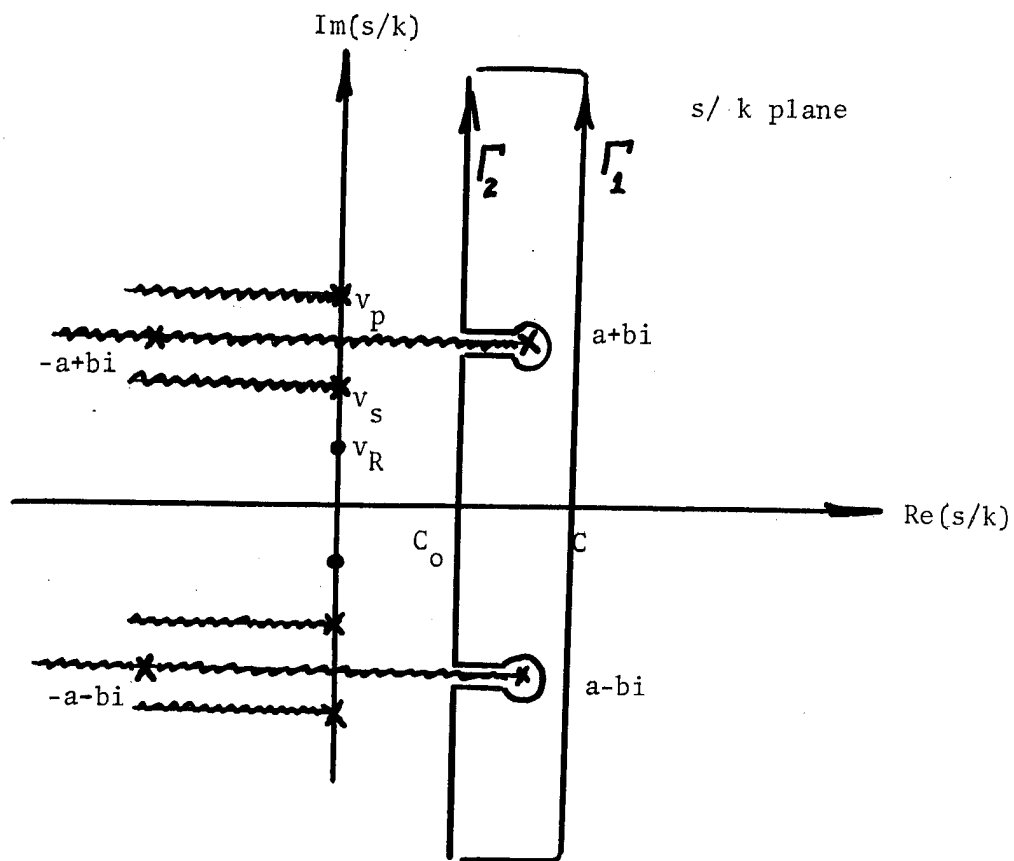


FIGURE 3.

Poles, Branch Points and Integration Contours in the complex Plane for the Numerical Solution.

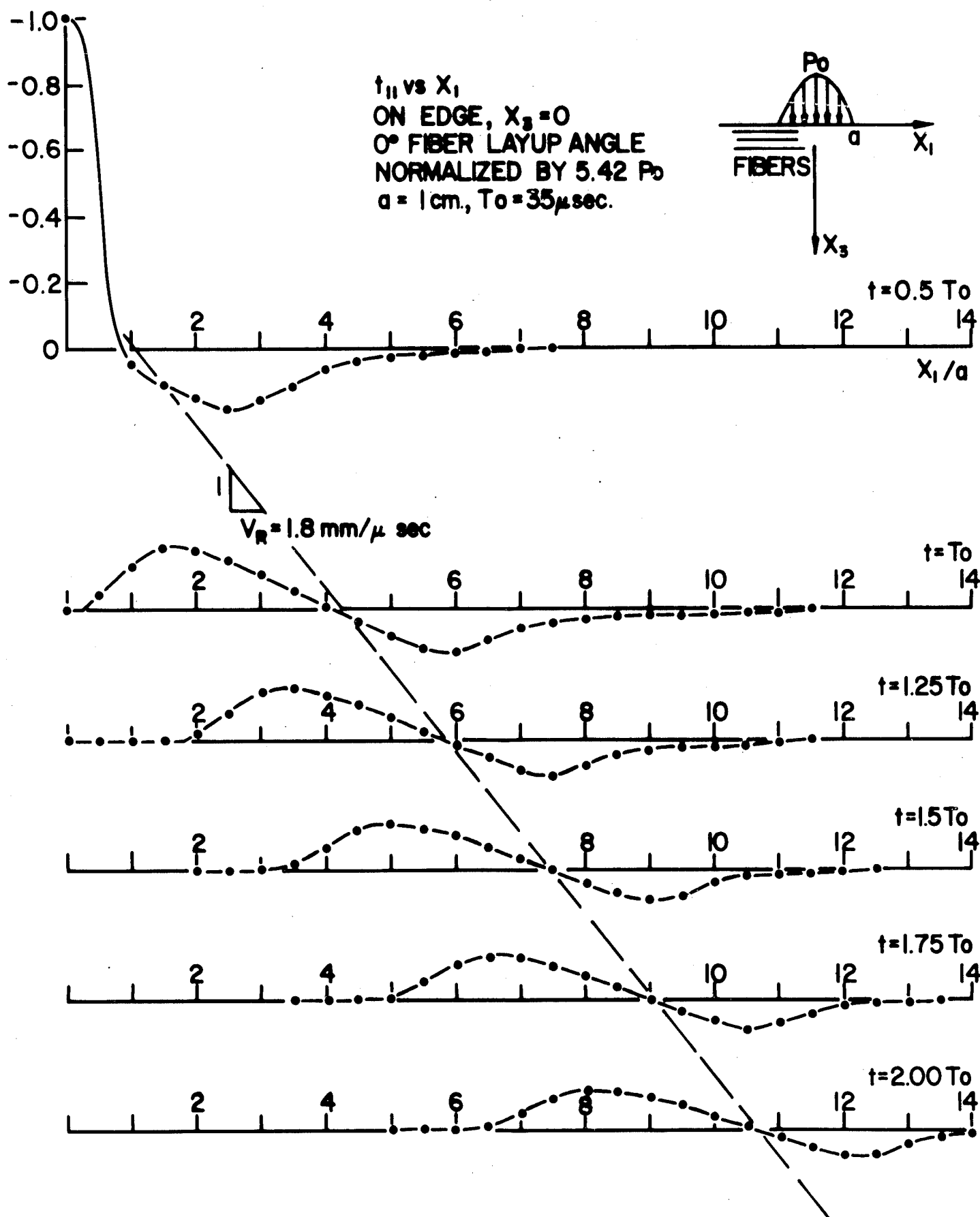


FIGURE 4

Edge Stress  $t_{11}$ , Surface Wave for 0° Fiber layup Angle: 55% graphite fiber/epoxy matrix composite

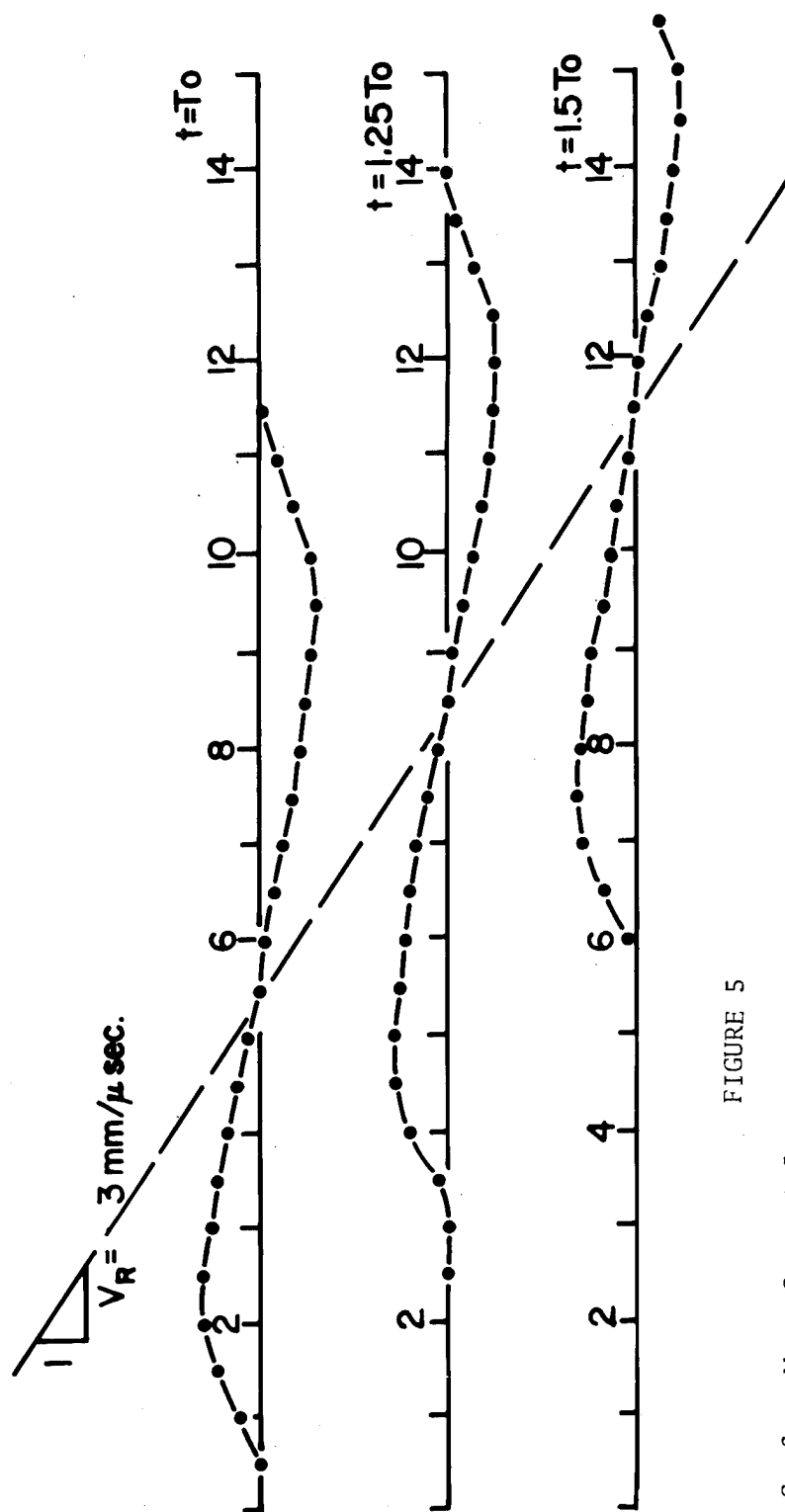
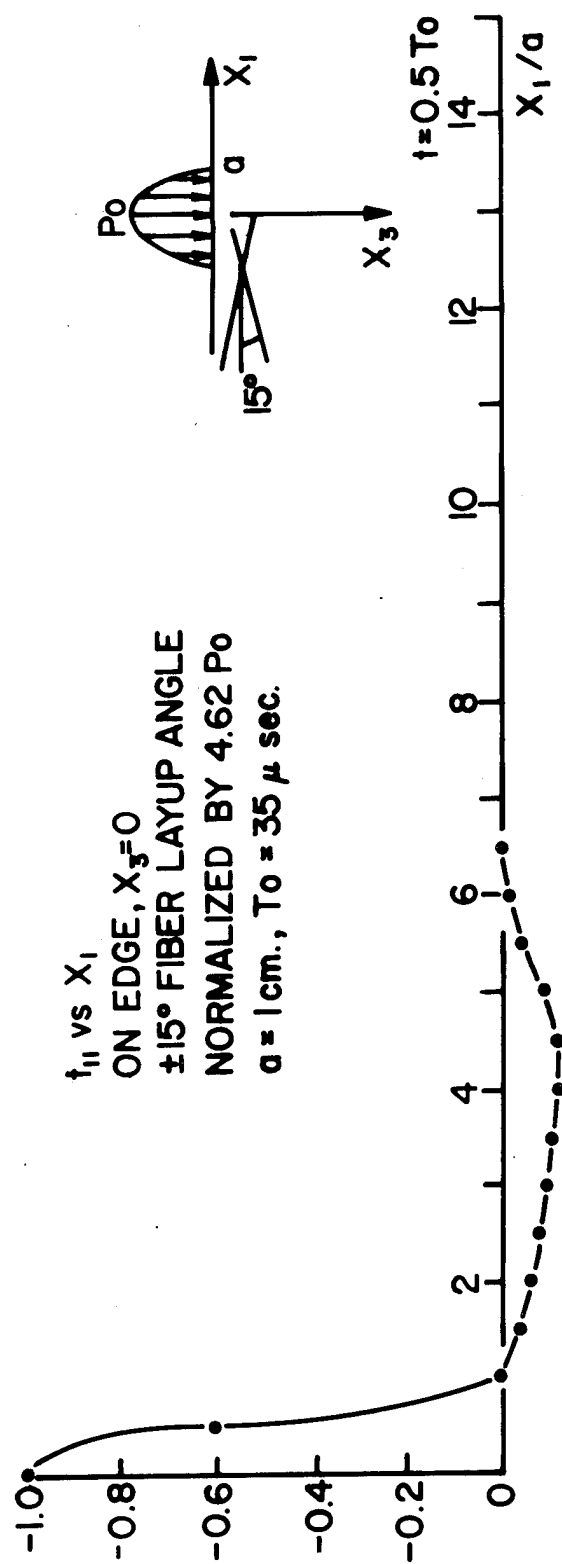


FIGURE 5

Edge Stress  $t_{11}$  Surface Wave for  $\pm 15^\circ$  Fiber Layout Angle: 55% Graphite fiber/epoxy matrix composite.

TIME, DT= 0.4366E-05

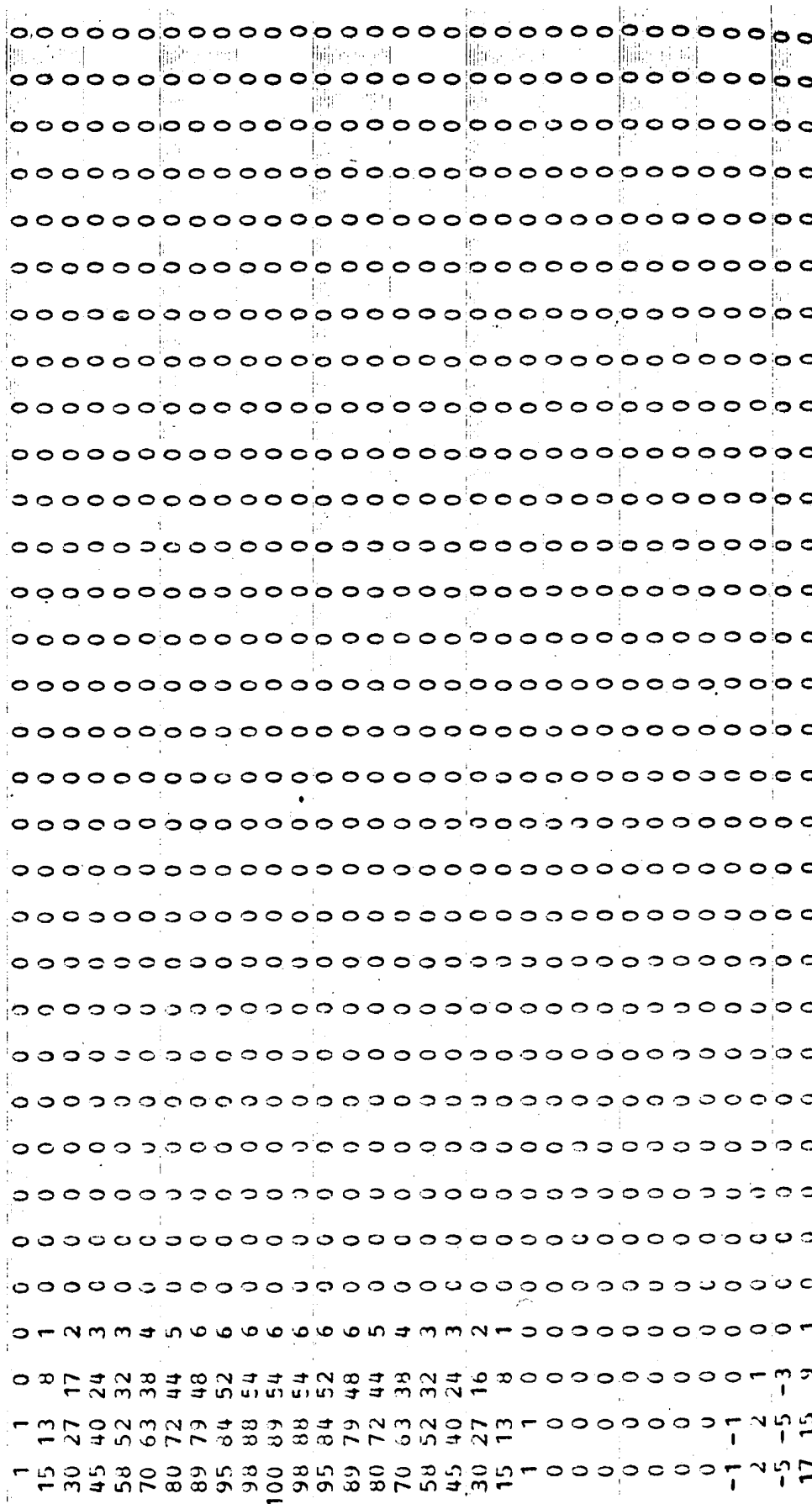


FIGURE 6

Computer Map of Edge Impact Pressure  $t_{33}$  Impact Time 87  $\mu$ sec.



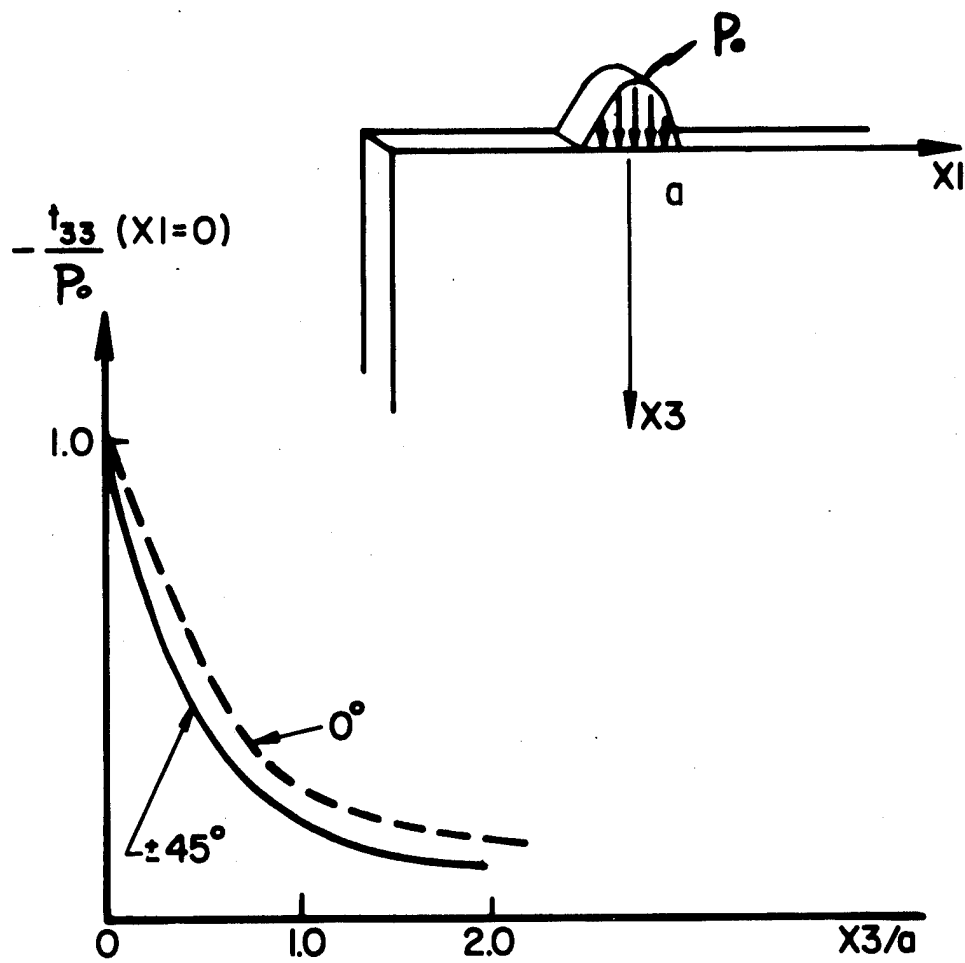


FIGURE 8  
Decrease of Stress  $t_{33}$  with Distance from the Impact Edge.

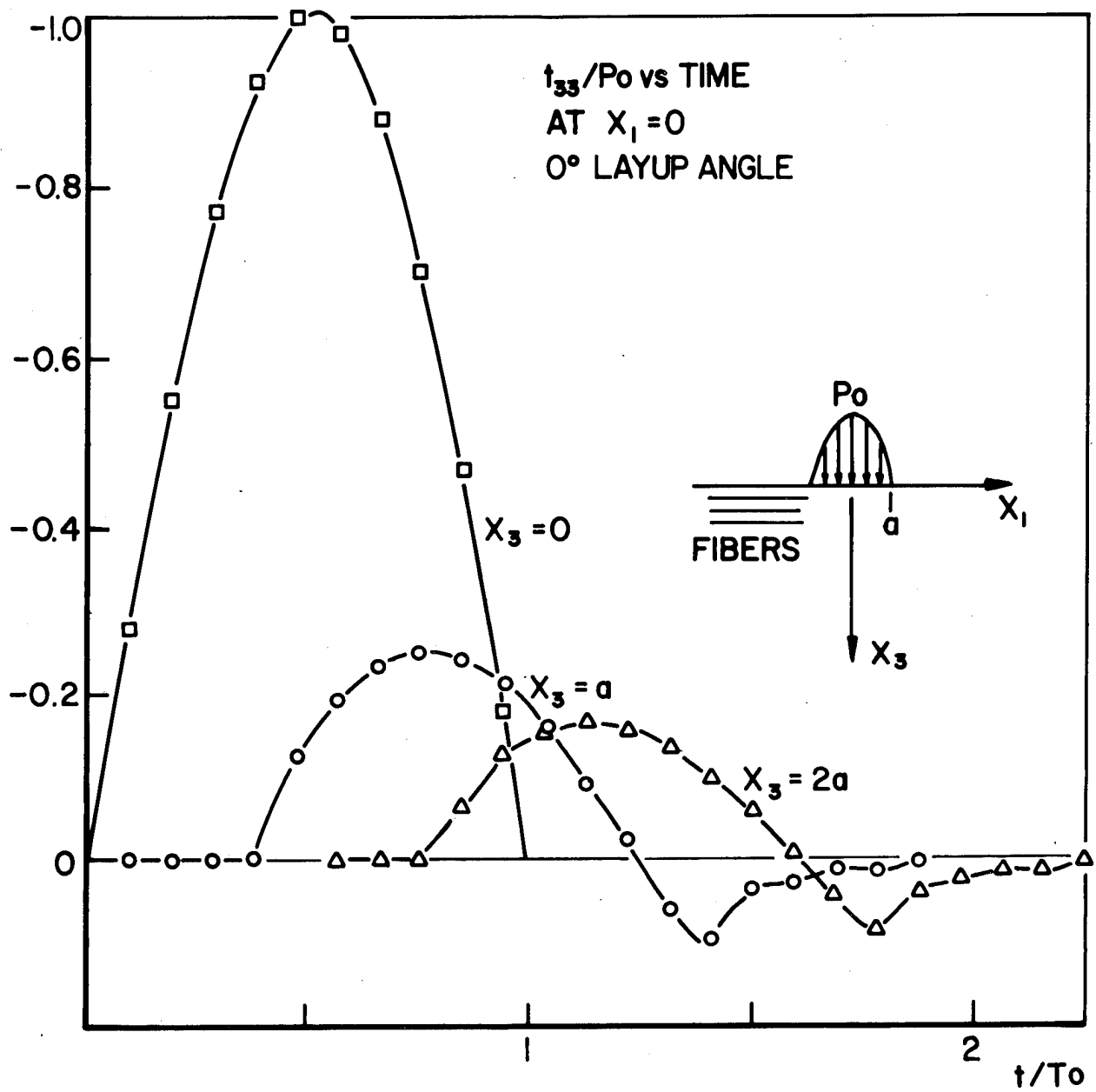


FIGURE 9

Stress  $t_{33}$  versus Time at Different Distances from the Edge under the Contact Point.

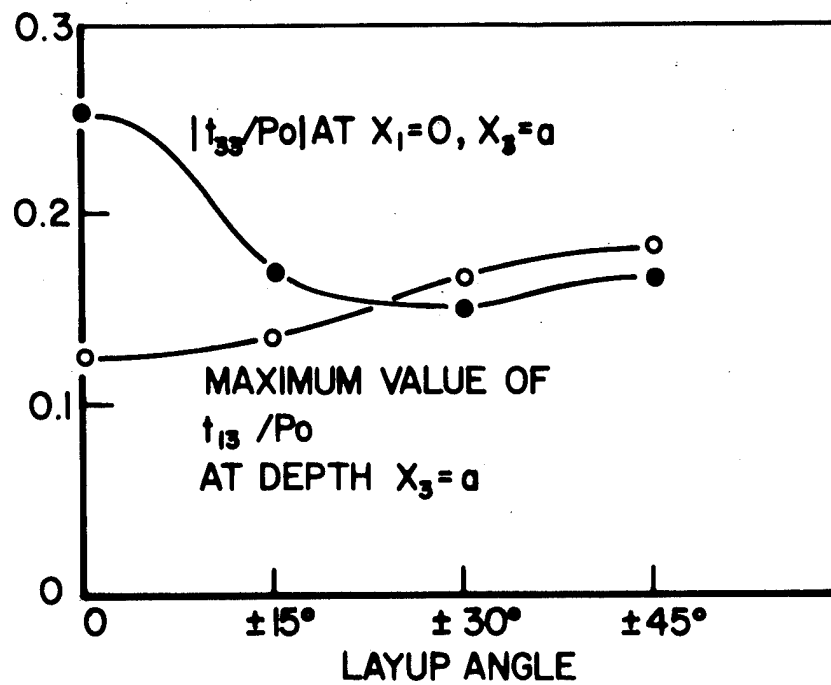
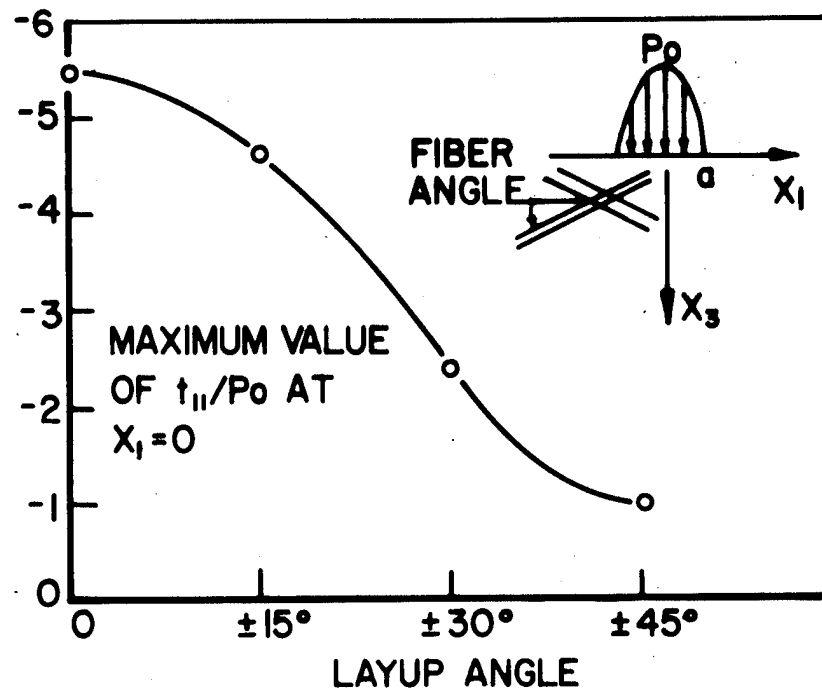


FIGURE 10

Maximum Stresses versus Layup angle for 55% Graphite Fiber/epoxy Matrix Composite.



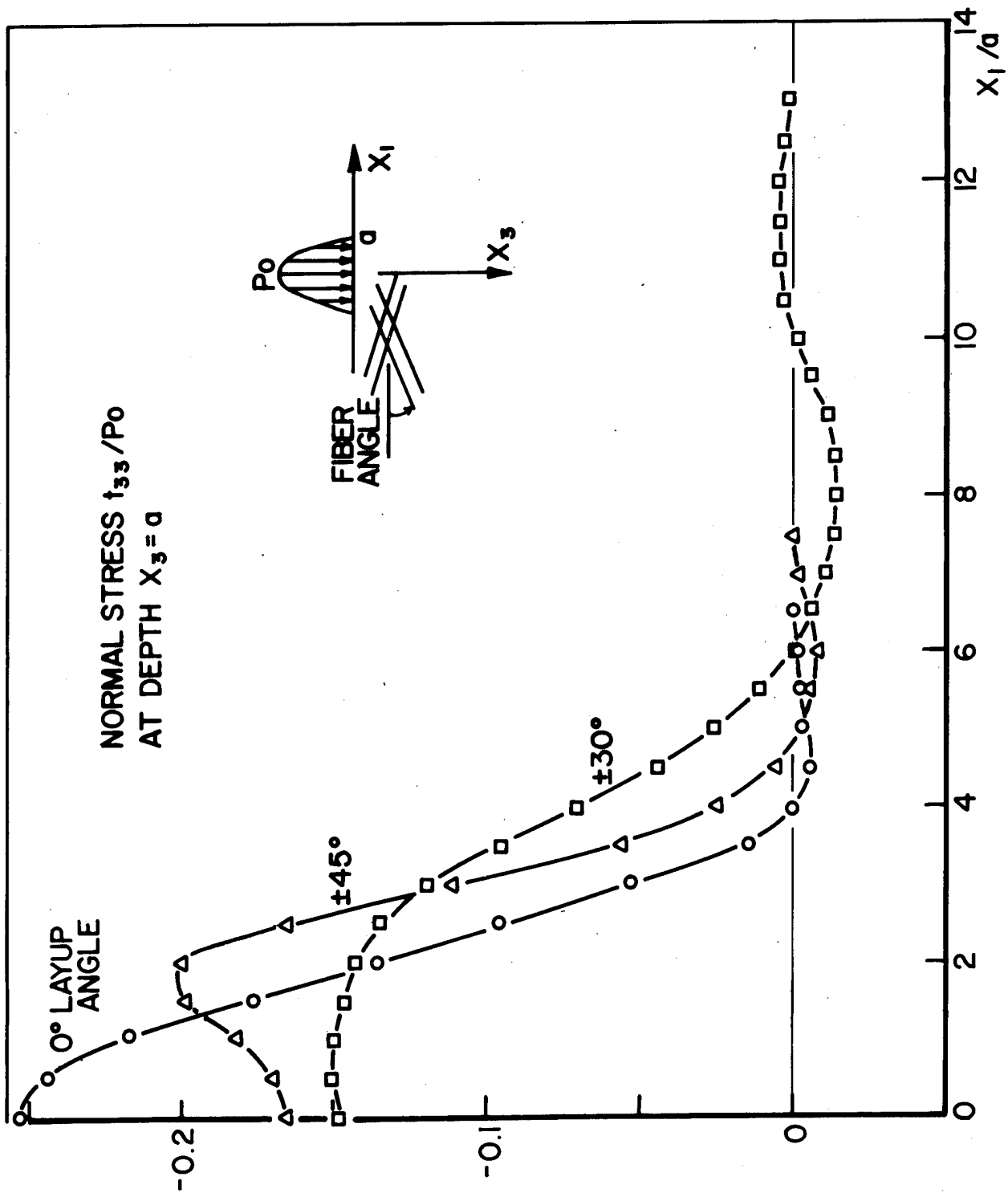
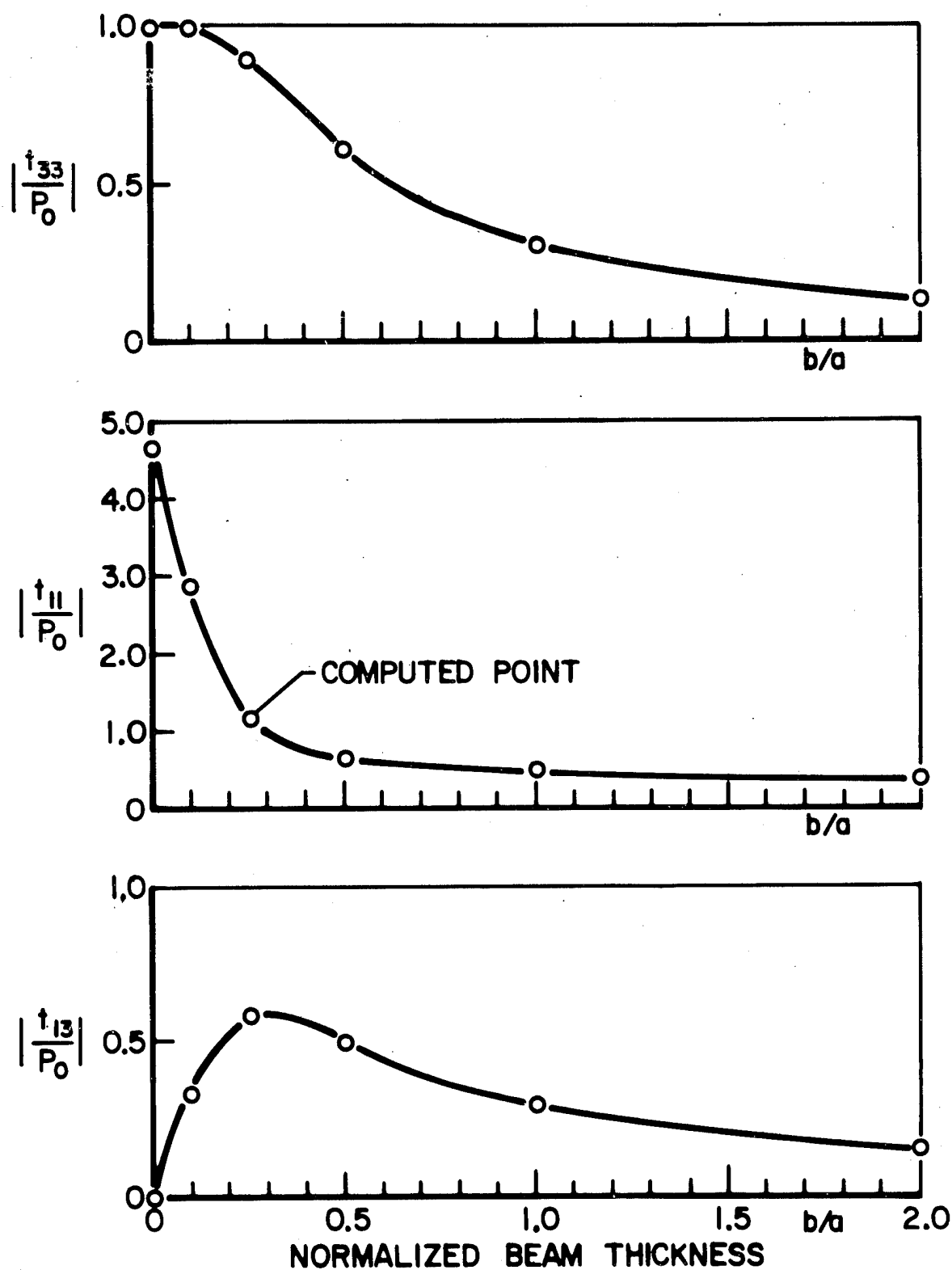


FIGURE 11

Distribution of Stress  $t_{33}$  Along the Edge for Fiber Layup angles  $0^\circ$ ,  $\pm 30^\circ$ ,  $\pm 45^\circ$ .

NORMALIZED STRESSES IN COMPOSITE  
AT STRIP-EDGE INTERFACE

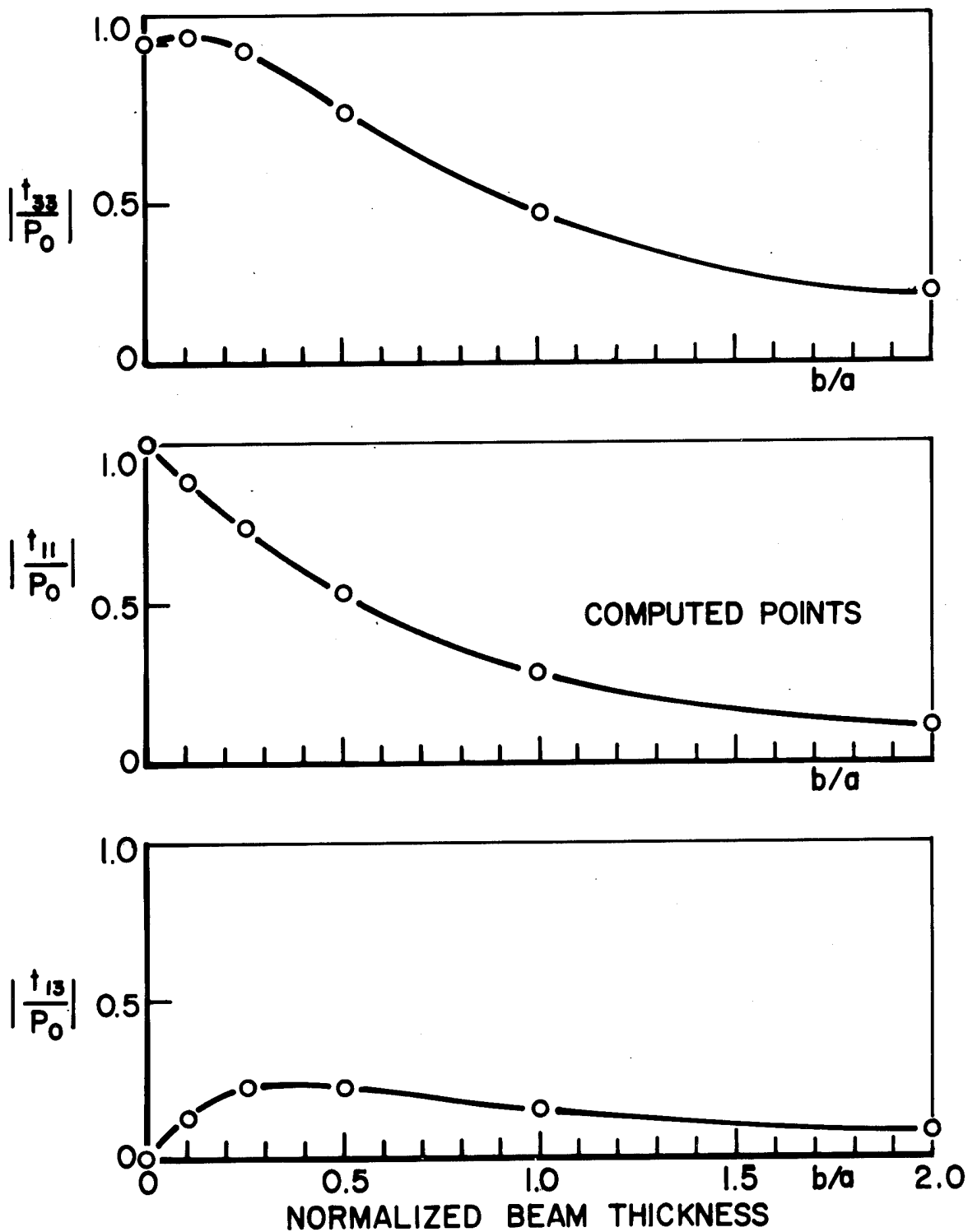


DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX  
 $\pm 15^\circ$  LAYUP - STEEL STRIP

FIGURE 12a

Effect of Edge Strip Thickness on Interface Stresses.

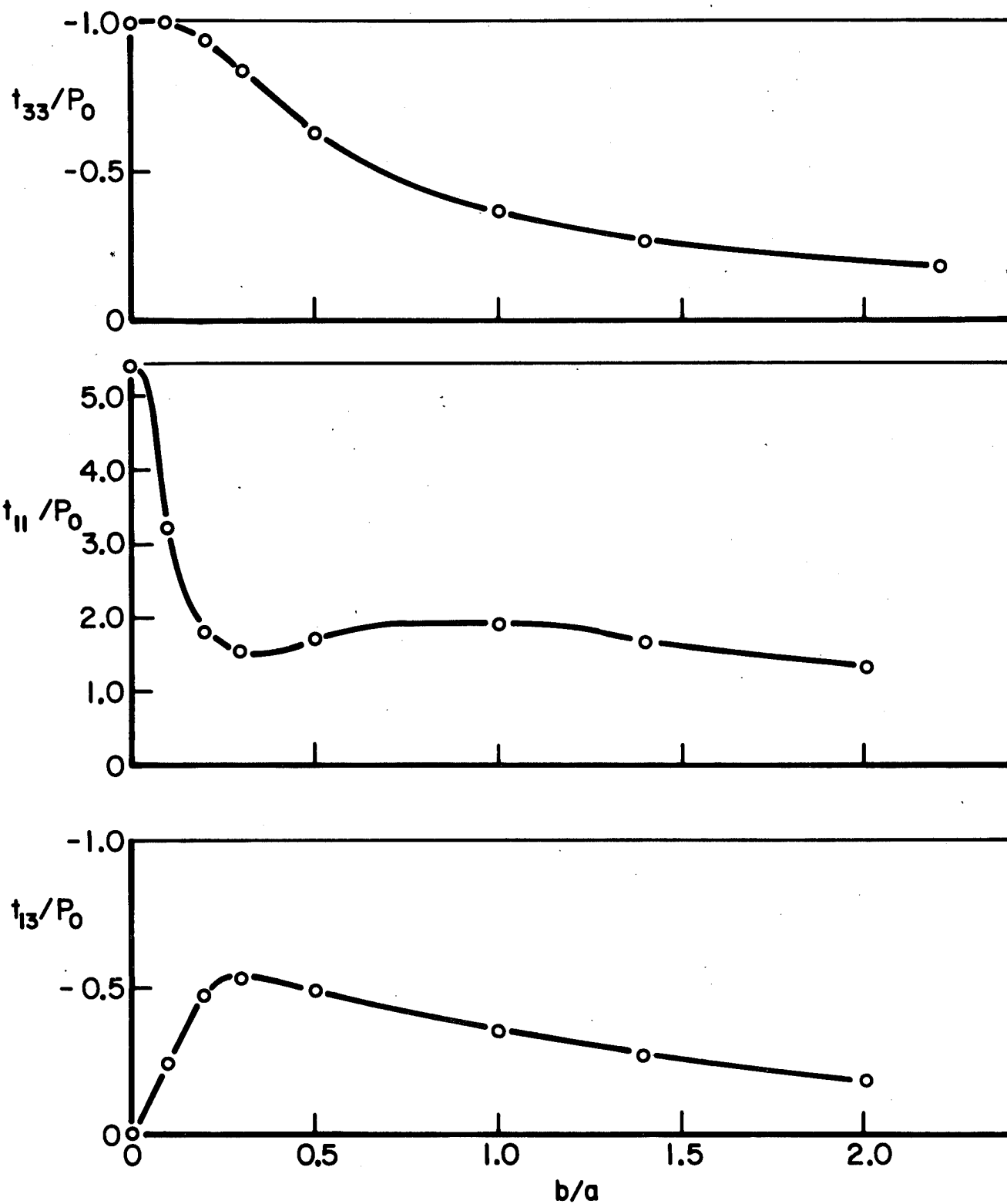
NORMALIZED STRESSES IN COMPOSITE  
AT STRIP-EDGE INTERFACE



DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX  
± 45° LAYUP ANGLE - STEEL STRIP

FIGURE 12b

Effect of Edge Strip Thickness on Interface Stresses.



0° LAYUP ANGLE ALUMINUM STRIP

$X_3 = 0$  ,  $a = 1\text{cm}$  ,  $T_0 = 35\mu\text{sec}$

FIGURE 13

Effect of Edge Strip Thickness on Interface Stresses.

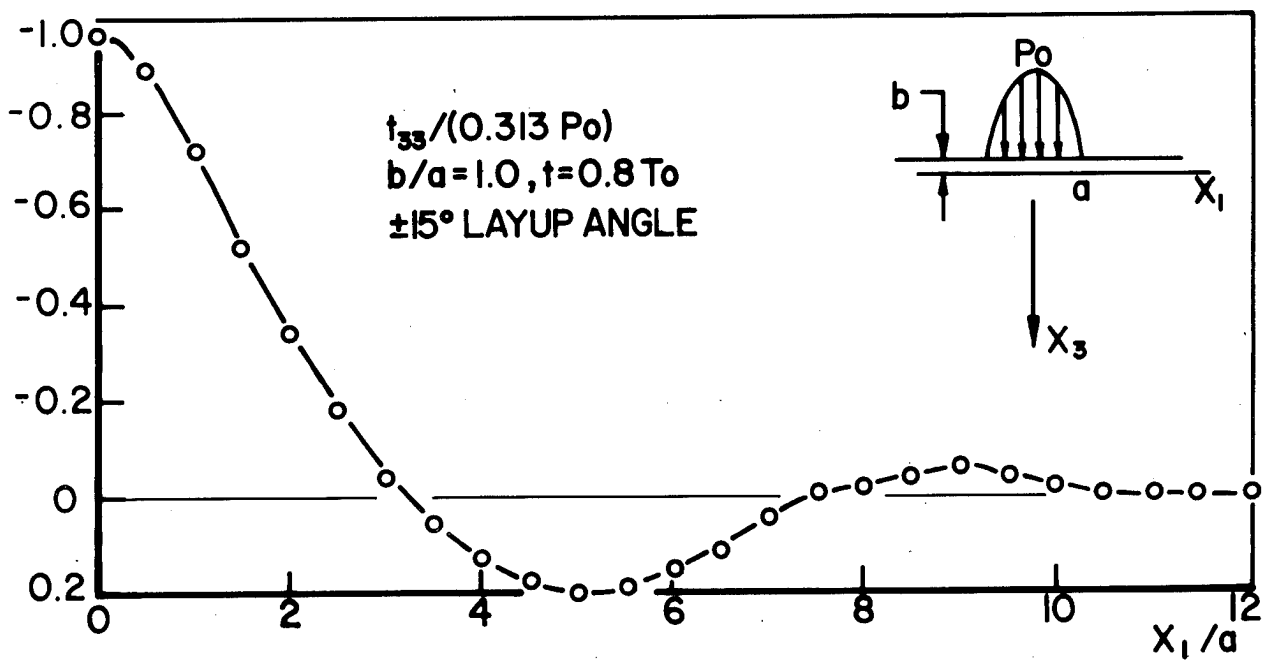
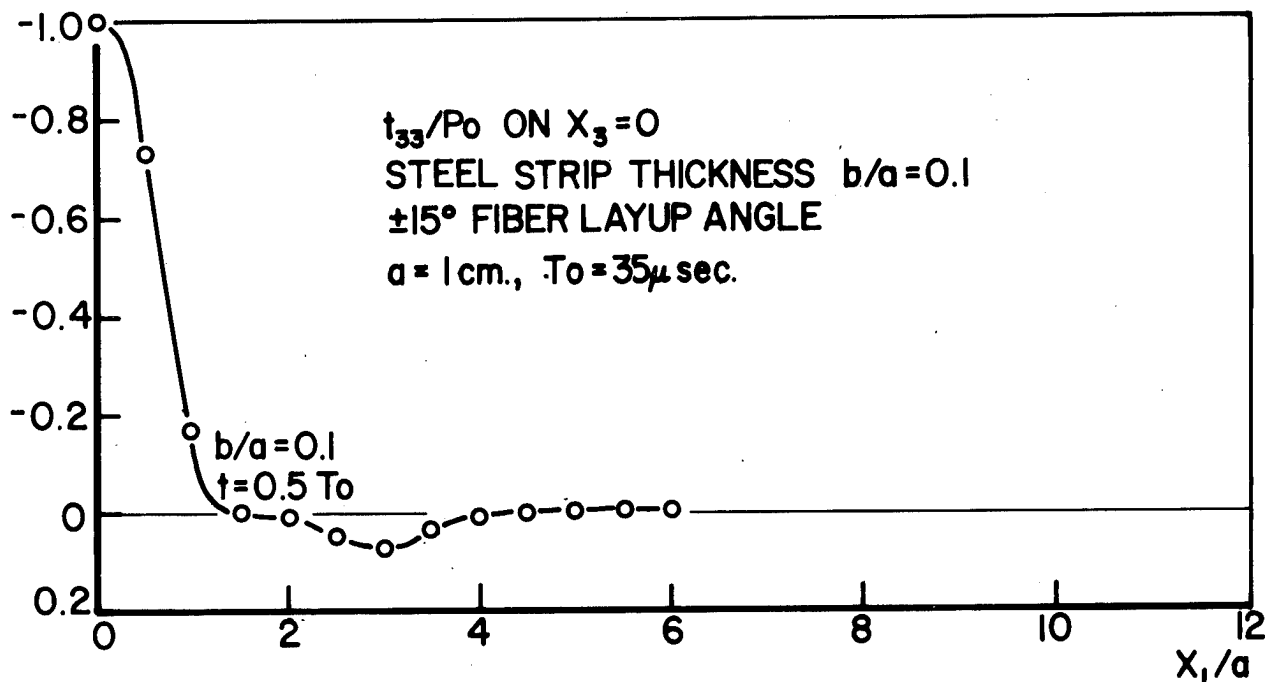


FIGURE 14  
 Effect of Edge Strip Thickness on Stress  $t_{33}$  Distribution Along the Edge.

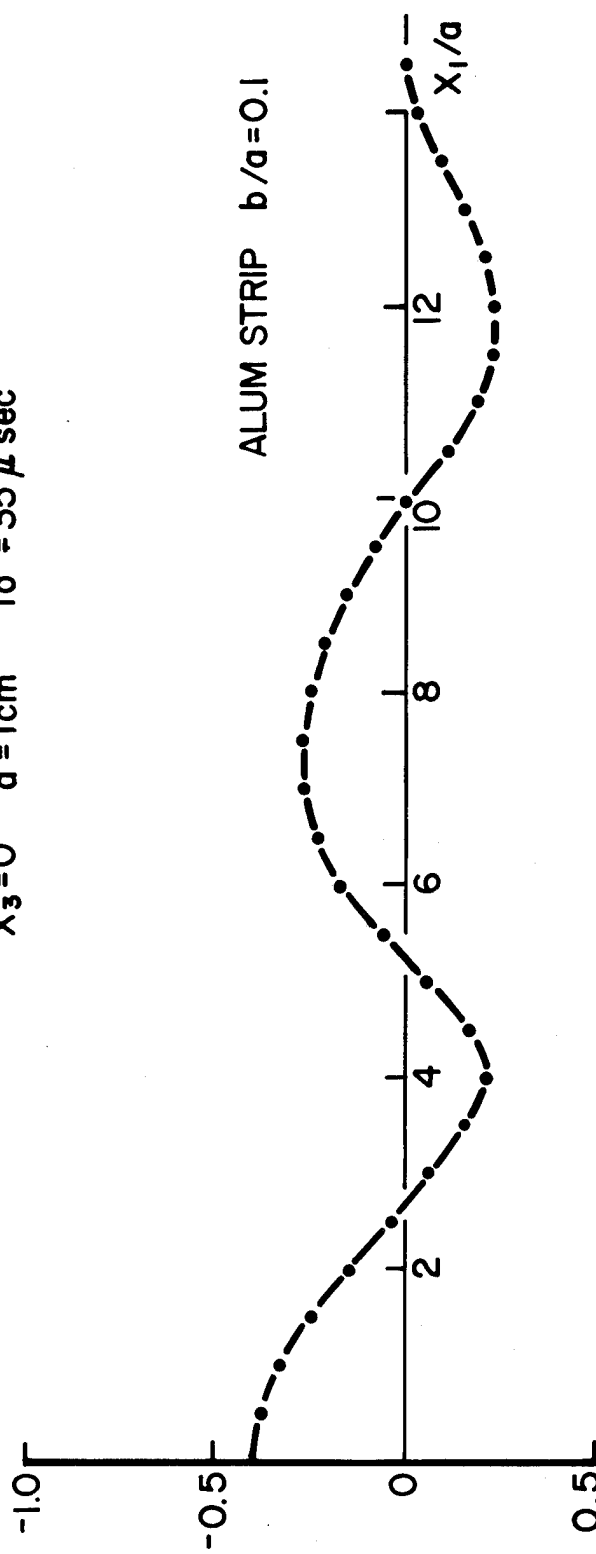
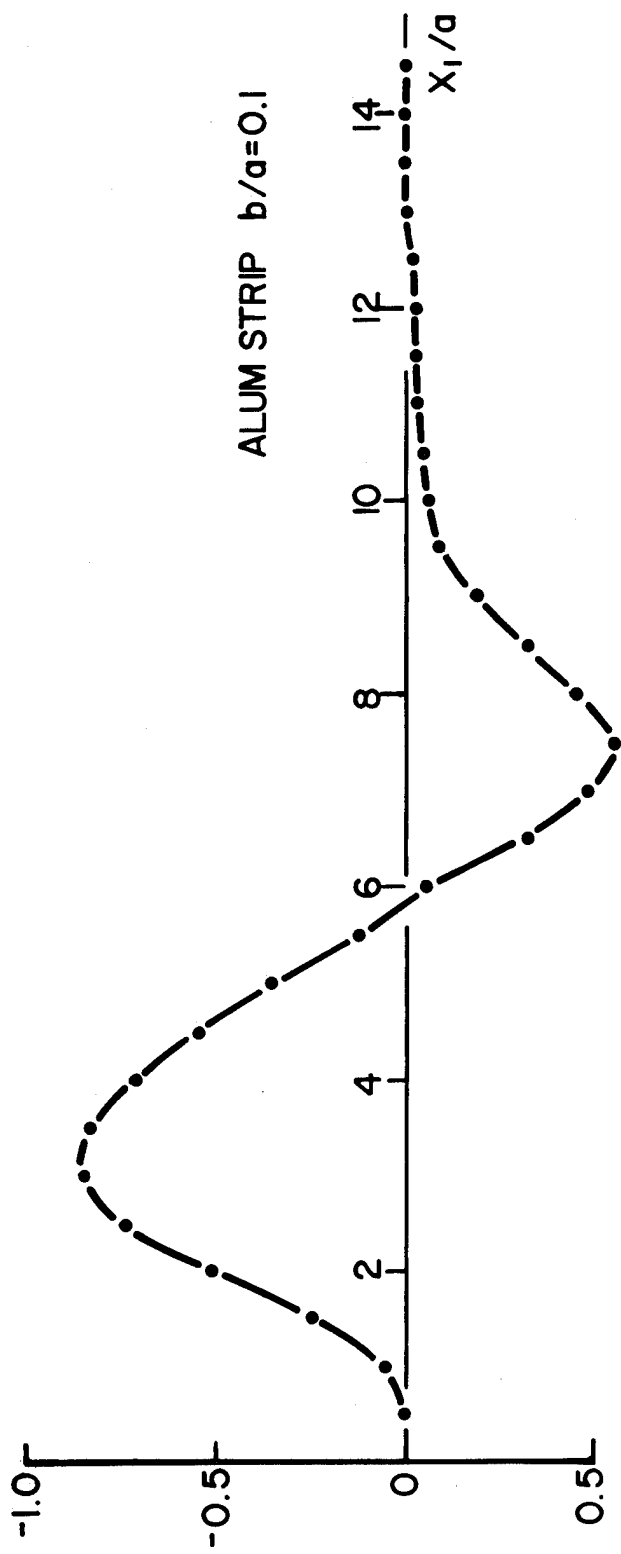


FIGURE 15

Effect of Edge Strip Thickness on the Rayleigh Edge Wave Shape.

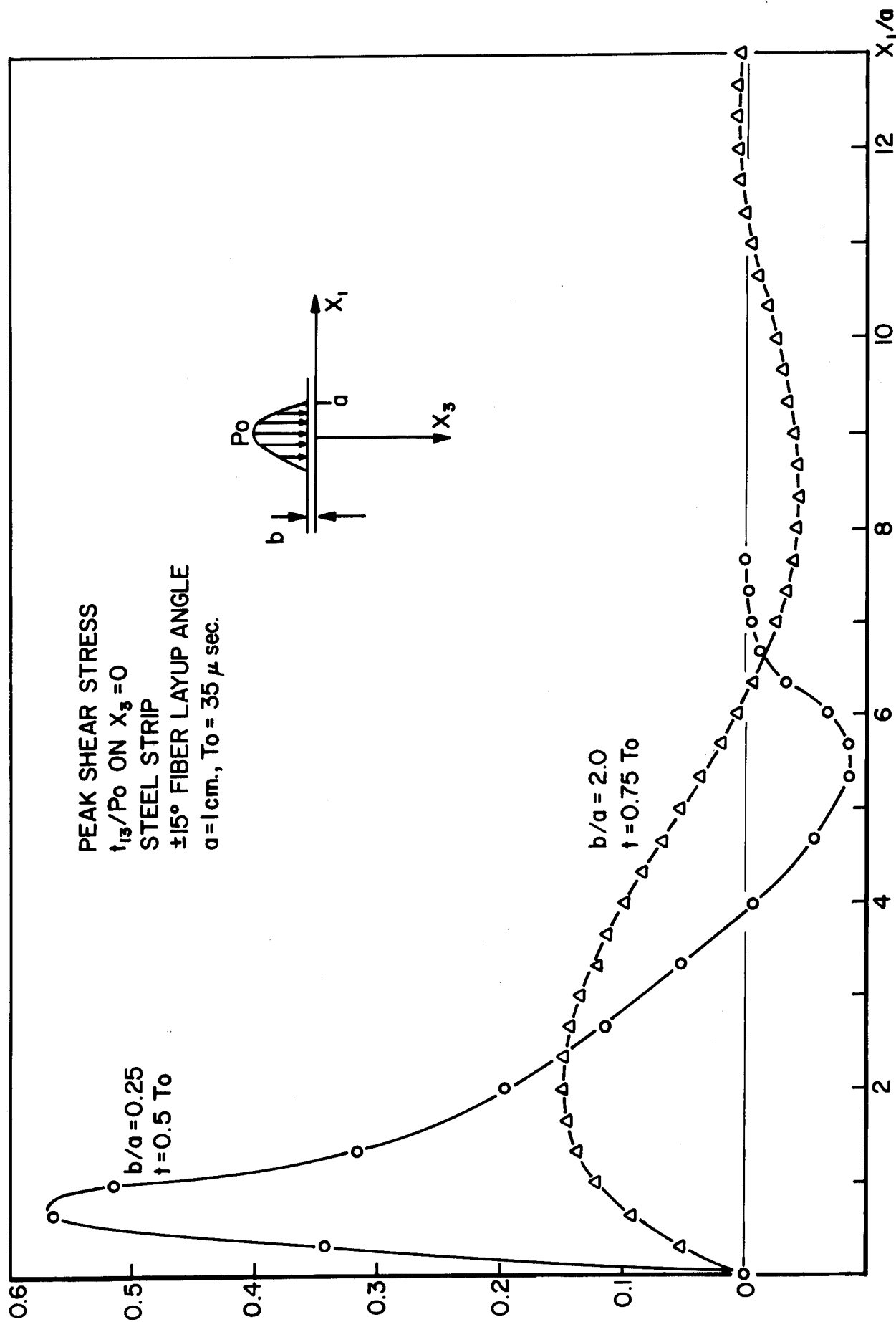


FIGURE 16

Effect of Edge Strip Thickness of Interface Shear Stress  $t_{13}$  Distribution Along the Edge.